PLASTICS GEARING

REVISED EDITION, JANUARY 1994

William McKinlay and Samuel D. Pierson

Published by:

ABA/PGT PUBLISHING

an affiliate of:

ABA-PGT Inc.
P.O. BOX 8270
Manchester, Connecticut 06040-0270

Library of Congress Catalog Card Number: 93-74822

Printed in the United States of America

INTRODUCTION

PLASTICS GEARING is a reference manual to which industry can turn for technical assistance in plastic gearing and design. It also outlines the services available from its publishers for those who wish to place any phase of design or manufacturing outside.

PLASTICS GEARING is intended to be a continuing publications. Copies are individually numbered and the names of recipients are kept on file. As new material is generated, it will either be forwarded for insertion at no charge or the holder will be advised of its availability and cost.

If this publication does not answer your questions on

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AN INVITATION TO VISIT US

We would be delighted to show you our facilities and discuss your specific requirements. We believe a visit to our plant will help you resolve a number of questions you may have concerning precision plastics gearing.

Dedicated to the memory of

WILLIAM McKINLAY

a good friend and associate
whose untiring efforts have contributed
significantly to the technology
of plastics gearing

the publishers
August 16, 1976

ACCURATE MOLDED

PLASTIC GEARS

BY
WILLIAM McKINLAY
AND
SAMUEL D. PIERSON

•

Drawings By
W. HAROLD STOCKBURGER

REVISED EDITION

ABA-PGT INC

INTRODUCTION TO THE FIRST EDITION

The best possible gear for any given application is the least expensive gear that will do the job required of it. Today, that best possible gear may very well be molded plastic gear. Better materials, advances in the design and construction of molding dies, the advent of small, high speed injection molding machines and improved molding techniques are making available plastic gears that match the best cut metal gears in accuracy and performance.

A molded plastic gear and a machined metal gear, made to the same specifications, are dimensionally identical, but the methods employed in their manufacture differ widely. This book is intended to be the first step towards closing the gap between gear engineering and plastics technology. It is based upon a background of many years of experience in the design and manufacture of gears and gear molding dies.

The contents take the form of answers to questions put to the authors by purchasing agents and engineers who were either faced with the procuring of plastic gears for the first time, or were unsatisfied with the quality of previous purchases. The authors trust the information will prove of value to all who are interested in this field.

William McKinlay
Samuel D. Pierson

Manchester, Conn. 1967

INTRODUCTION TO THE SECOND EDITION

All of the topics of this work have been subject to thoughtful research and searching thought since the original publication in 1967. There are details that require minor change and there is much new knowledge to add.

There has been considerable temptation to abandon this work in favor of a new format to bring everything up to date as best the authors can. Yet the authors are pleased to be continually reminded that many still consider "Accurate Molded Plastic Gears" to be a useful work and there is still a place for this "first step towards closing the gap between gear engineering and plastics technology".

To reconcile this conflict, the authors have retained the present format – revising it to be revelant in 1976. This is for those who wish to have no more than a general understanding of the subject.

William McKinlay has written a new and comprehensive design manual, "A System For Involute Spur & Helical Gears Molded of the Plastics" which is presented as Section B of "Plastics Gearing". Section B is intended for the use of engineers who may be required to design plastics gears for specific applications.

William McKinlay
Samuel D. Pierson

Manchester, Conn. 1976

INTRODUCTION TO THE REVISED EDITION

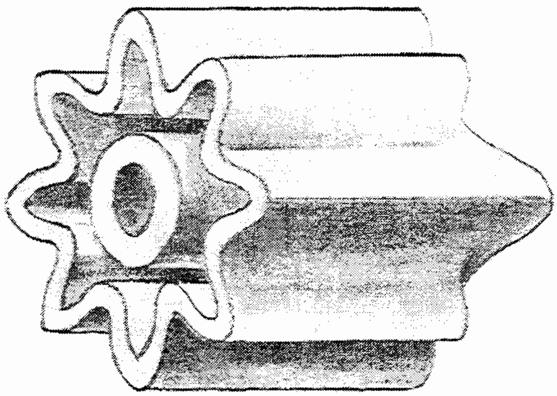
With the passage of years, William McKinlay's work remains valid. There are few – mostly insignificant – changes included in this edition. What better tribute could there be to this man's mastery of plastics gear engineering.

Samuel D. Pierson

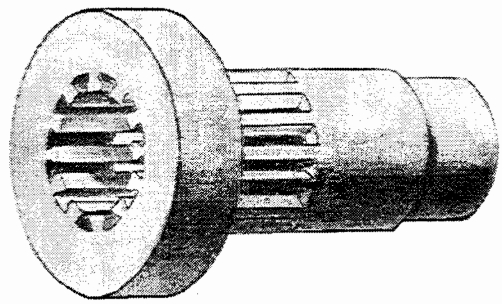
Manchester, Connecticut 1994

ACCURATE MOLDED PLASTIC GEARS

CONTENTS



CORED 10 D.P. PINION: ACETAL.



120 D.P. PINION, TEETH SUSPENDED BETWEEN FLANGES: NYLON

ACKNOWLEDGEMENTS

The authors have drawn freely upon the experience of ABA-PGT inc. in creating a wide variety of gear-molding dies. These dies are molding millions of gears of all types and pitches for use in mechanisms ranging from heavy drives to the most delicate of instruments.

The illustrations throughout the text are of gears similar to those being produced in ABA molding dies for instruments, counting devices, cameras, automotive, appliances, printers, plotters, meters and toys. Artistic liberties have been taken with the illustrations to protect the integrity of proprietary designs.

Reference to published texts are included in the subject matter.

WHAT PLASTICS ARE USED FOR GEARS?

Gears can be molded of many engineering plastics. In spite of the introduction of many new materials, the majority of applications still call for nylons and acetals. In special circumstances, acrylonitrile-butadiene-styrenes (ABS), polycarbonates, polysulfones, phenylene-oxides, polyurethanes, and thermoplastic polyesters can also be considered. All can be obtained in various grades and in filled varieties.

A filled plastic is one to which a material has been added to improve its mechanical properties. The additives normally used in gear plastics are glass, polytetraflouroethylene (PTFE), silicones, and molybdenum disulphide. Glass is added in the form of short hairlike fibers, miniscule beads, or a fine milled powder. Fibers increase the tensile strength of the molded part just as steel rods reinforce a concrete structure. The presence of fibers, beads or powder causes the part to be more dimensionally stable, but beads or powders do not contribute strength as does fibers. Glass fiber reinforcement can as much as double the tensile strength of a basic material and any type of glass can reduce the thermal expansion of the basic material to as little as one third of the original value.

Carbon fiber is often considered as an additive to increase strength beyond that achievable with glass fibers. It is also claimed to increase wear characteristics. However, these benefits come with some cost. Gear accuracy suffers with the addition of fibers (glass or carbon), there is increased wear on production equipment, and material is more costly. These factors must be considered when designing with carbon filled material.

Molybdenum disulphide, PTFE, PFPE and silicones act as built-in lubricants and make for increased wear resistance. Plastics containing both glass and lubricants are popular for gears.

Selection may be based on financial cost, as well as mechanical or chemical properties. Materials vary not only in basic cost, but in tooling requirements and processing time.

The foregoing merely scratches the surface of the subject of materials. New resins and additives are on the horizon that may offer new options for tomorrow's gear designers. To obtain the most current material data, refer to available sources, such as Modern Plastics Encyclopedia and material suppliers. Data sheets tabulating the mechanical and chemical properties of the plastics and other useful information, can be obtained from them.

WHAT DETERMINES WHICH PLASTIC TO SPECIFY FOR A GIVEN GEAR?

As with any other material, the choice of the plastic is governed by the size and nature of the load to be transmitted, the speed, the life required, the environment in which the gear will operate, the type of lubrication, and the degree of precision necessary.

Because no two gear applications are alike, and because of the wide range of plastics available, it is beyond the scope of this chapter to deal with the subject on any but general terms. The equations that follow, used in conjunction with the accompanying tables, will help to determine which, if any, of the plastics are viable materials in terms of the strength required. The equations are simple variations of the Lewis formula, and assume the use of standard tooth forms. The answers they give are conservative, and should be so regarded; but are of sufficient accuracy to enable a decision to be made whether a further study in depth is warranted. Such a study will involve a requirement that the gear be designed for the specific application under review. How this is accomplished is the subject matter of the sections to follow.

WHEN USING THE PGT BALANCED TOOTH STRENGTH SYSTEM, REFER TO CHAPTER 11 FOR A MORE ACCURATE ANALYSIS.

Spur gearing (External and Internal)

$$HP = \frac{S_s F Y V}{55(600 + V) P C_s}$$

Helical Gearing (External and Internal)

$$HP = \frac{S_s F Y V}{423(78 + \sqrt{V}) P_n C_s}$$

Straight Bevel Gearing

$$HP = \frac{S_s F Y V (C - F)}{55(600 + V) P C C_s}$$

S_s = safe stress. Table 1
F = face width in inches
Y = tooth form factor. Table 2
V = velocity in feet per minute at pitch circle diameter
P = diametral pitch
P_n = normal diametral pitch
C_s = service factor. Table 3
C = outer cone distance

Table 1 SAFE STRESS

Plastic	Safe Stress	
	Unfilled	Glass-reinforced
ABS	3,000	6,000
Acetal	5,000	7,000
Nylon	6,000	12,000
Polycarbonate	6,000	9,000
Polyester	3,500	8,000
Polyurethane	2,500	

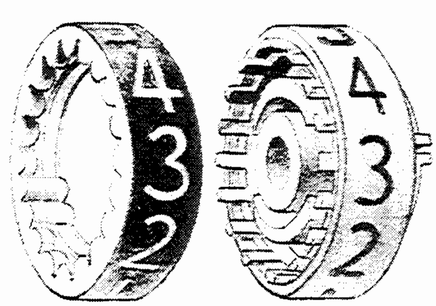
ACCURATE MOLDED PLASTIC GEARS

The figures for safe stress in Table 1 allow for a moderate temperature increase and assume some initial lubrication. The values given to the safe stress of the glass-reinforced plastics should be used with discretion. The glass-reinforced varieties have qualities that make them superior to the unfilled plastics for certain gear applications; but for other applications their greater strengths may be more apparent than real.

Table 2. TOOTH FORM FACTOR Y

Number of Teeth	14½° Involute or Cycloidal	20° Full Depth Involute	20° Stub Tooth Involute	20° Internal Full Depth	
				Pinion	Gear
12	0.210	0.245	0.311	0.327	-
13	0.220	0.261	0.324	0.327	-
14	0.226	0.276	0.339	0.330	-
15	0.236	0.289	0.348	0.330	-
16	0.242	0.295	0.361	0.333	-
17	0.251	0.302	0.367	0.342	-
18	0.261	0.308	0.377	0.349	-
19	0.273	0.314	0.386	0.358	-
20	0.283	0.320	0.393	0.364	-
21	0.289	0.327	0.399	0.371	-
22	0.292	0.330	0.405	0.374	-
24	0.298	0.336	0.415	0.383	-
26	0.307	0.346	0.424	0.393	-
28	0.314	0.352	0.430	0.399	0.691
30	0.320	0.358	0.437	0.405	0.679
34	0.327	0.371	0.446	0.415	0.660
38	0.336	0.383	0.456	0.424	0.644
43	0.346	0.396	0.462	0.430	0.628
50	0.352	0.408	0.474	0.437	0.613
60	0.358	0.421	0.484	0.446	0.597
75	0.364	0.434	0.496	0.452	0.581
100	0.371	0.446	0.506	0.462	0.565
150	0.377	0.459	0.518	0.468	0.550
300	0.383	0.471	0.534	0.478	0.534
Rack	0.390	0.484	0.550	-	-

For bevel gearing, multiply number of teeth in gear by the secant of the pitch angle and use answers in Table 2. For example, if a 20° PA bevel gear has 40 teeth and a pitch angle of 58°, $40 \times \text{secant } 58° = 40 \times 1.88708 = 75$, and $Y = .434$.



NUMBER DRUMS, 38 D.P. INTERNAL AND EXTERNAL GEARS: ACETAL

Table 3. SERVICE FACTOR

Type of Service

Type of Load	8-10 hours per day	24 hours per day	Intermittent 3 hours per day	Occasional ½ hour per day
Steady	1.00	1.25	0.80	0.50
Light shock	1.25	1.50	1.00	0.80
Medium shock	1.50	1.75	1.25	1.00
Heavy shock	1.75	2.00	1.50	1.25

At this point it might be helpful to work through an example. Assume that it is required to choose a plastic for a spur gear which is to transmit ⅛ HP at 350 R.P.M. The gear will run 8 hours per day and the load is steady. The gear has the following data:

Number of teeth	75
Diametral pitch	32
Pressure angle	20°
Pitch diameter	2.34375
Face width	.375

$$HP = \frac{S_s F Y V}{55(600 + V) P C_s}$$

or

$$S_s = \frac{55(600 + V) P C_s HP}{F Y V}$$

$P = 32$ $\quad D = 2.34375$
$C_s = 1.0$ $\quad RPM = 350$
$HP = .125$ $\quad Y = .434$
$F = .375$

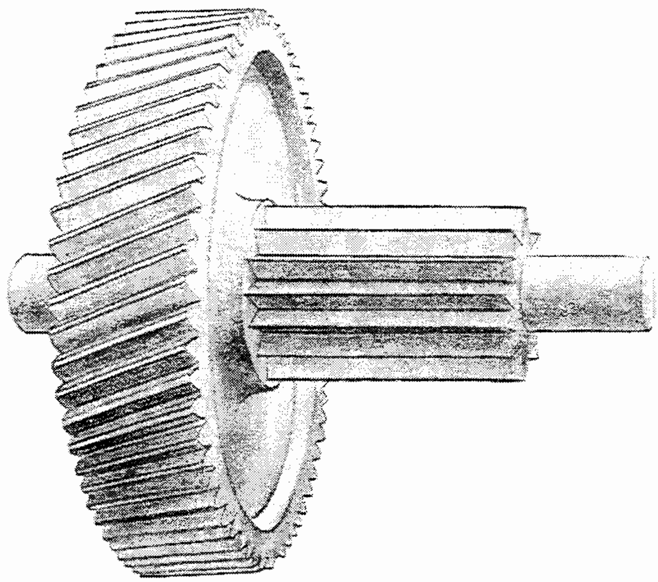
$$V = \frac{RPM \times \pi \times D}{12}$$

$$V = \frac{350 \times 3.1415926 \times 2.34375}{12}$$

$= 215$ f.p.m.

$$S_s = \frac{55(600 + 215)\,32 \times 1.00 \times .125}{.375 \times .434 \times 215}$$

$= 5{,}124$ P.S.I.

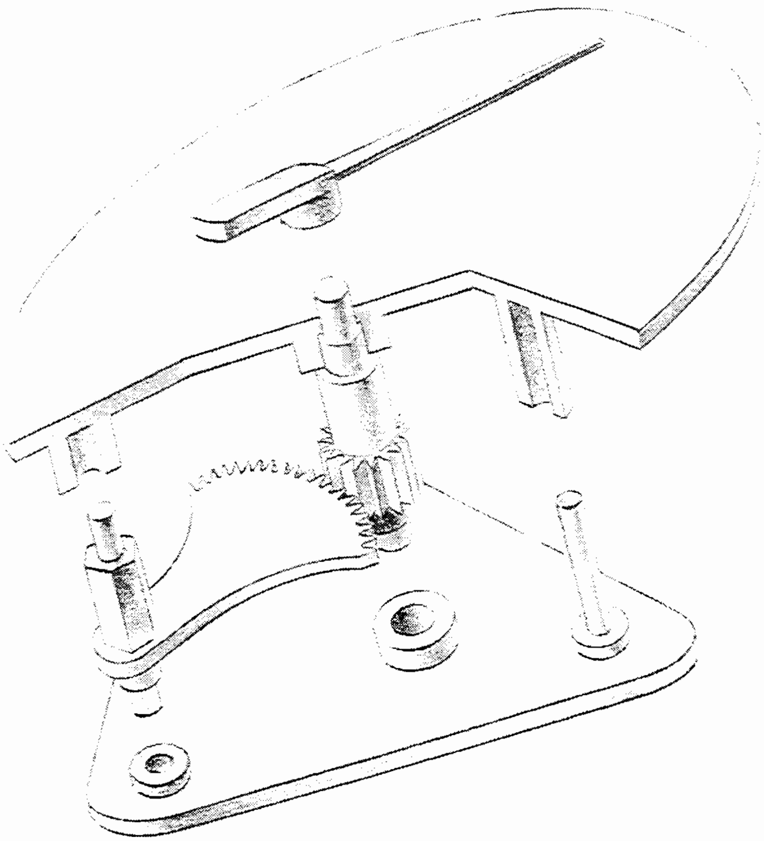


32 N.D.P. HELICAL GEAR AND 32 D.P. PINION: MOLYBDENUM DISULPHIDE - FILLED NYLON

Referring to Table 1 it would appear that the gear could be molded of a number of plastics. But now the physical and chemical properties of these plastics must be studied in relation to the environment in which the gear is to operate. The strength of the plastics fall off to a greater or lesser degree with increase in temperature; not all plastics are resistant to the action of certain liquids, including some lubricants; there are a few which degrade when exposed for long periods to direct sunlight; some are more dimensionally stable than others; and resistance to wear varies from one plastic to another.

ACCURATE MOLDED PLASTIC GEARS

As will be apparent, the ultimate choice of a plastic demands a close study of the project, preferably with the help of the plastic manufacturer and the molder. The recognized gear molders can be of considerable assistance as they have many case histories on which to draw in making their recommendations. Price, of course, is an important factor in arriving at the final choice. This would appear to be self evident, but it is not always given sufficient consideration.



INSTRUMENT ASSEMBLY, 120 D.P. SECTOR GEAR: ACETAL, 120 D.P. PINION: NYLON, HOUSING: POLYCARBONATE.

ACCURATE MOLDED PLASTIC GEARS

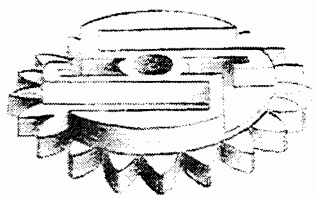
WHAT PROPERTIES PECULIAR TO PLASTICS REQUIRE CONSIDERATION IN THE DESIGN OF A PLASTIC GEAR?

Nothing has as yet been uncovered about the behavior of plastics in gearing that would indicate any necessity to depart from the principles of design established for gears of other materials. But the physical characteristics of plastics make it essential to adhere to these principles more rigidly than would necessarily be the case in designing gears to be machined of the metals. The design of all plastics gearing should receive the close study normally reserved for metal gears in critical applications.

The coefficients of linear thermal expansion of plastics, particularly the unfilled varieties, are considerably greater than those of the metals. Whereas there are only isolated instances when it is necessary to take into consideration the expansion of metal gears with increase in temperature, it is necessary to calculate the amount by which plastic gears will expand at the highest temperature to which they will be subjected, and to provide sufficient backlash to prevent binding. Some plastics are hygroscopic. Gears of these materials will expand to some degree if exposed to moisture. This rarely poses a problem, but additional backlash should be allowed if the mechanism of which the gears are part may remain unused for long periods in a damp atmosphere.

The teeth of heavily loaded metal gears in critical drives are given a degree of tip relief to lessen the ill effects of deflection, and have full fillet root radii to reduce fatigue stresses. These modifications should be specified for the teeth of all plastic gears.

In designing a pair of gears, use is frequently made of what is known as the long-short addendum system. If the pinion has a small number of teeth, these teeth may be undercut. Undercutting can be eliminated by increasing the addendum of the pinion teeth and decreasing that of the gear teeth. Undercutting weakens teeth, causes undue wear and may obviate continuity of action. Applying the long-short addendum system in designing gears also decreases the amount of approach action that occurs as the teeth of mating gears are going through the initial stage of their contact. Approach action is more taxing in terms of wear than the recess action that takes place during the later stage of tooth contact. The elimination of undercutting and the reduction of approach action are particularly beneficial when the gears are of plastic.



36 D.P. PINION AND PAWL
POCKETS: ACETAL

The advantages of the molding process as a means of fashioning gears must also be realized. The designer is freed from many of the limitations imposed by the necessity to think in terms of what is possible in using machine tools to make the blanks, and standard hobs and cutters, in generators and shapers, to form the teeth. Not only can expensive machining and assembly operations be eliminated by designing the gear to be integral with other parts in the one molding, but desirable modifications to the standard tooth can be specified without increasing the price of the gear, and at little or no additional cost for the molding die.

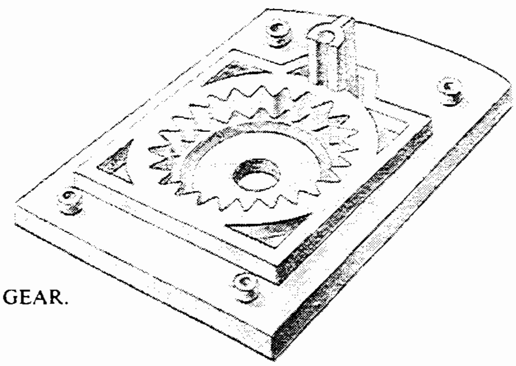
A number of designers have been quick to recognize and exploit these advantages. There are now being molded for use in instrument mechanisms gears of such complexity that their fabrication in metal would not be economically feasible. The various illustrations throughout this section indicate only some of the possibilities. In applying the molding process to the production of gears there is today little that can be ruled out as being impossible or impracticable. Consultation with a molding die maker before designs are made final may well result in considerable cost savings in the overall project by combining several features in a single molding.

HOW SHOULD PLASTIC GEARS BE DESIGNED?
HOW SHOULD THEIR SPECIFICATIONS BE WRITTEN?

If the first production samples of a machined gear prove to be for any reason unsatisfactory, effecting the necessary changes usually involves nothing more than a few adjustments to the settings of the machine used to cut the teeth. The first samples of a molded plastic gear are obtained only after there has been built a molding die on which has been expended many man-hours of highly skilled labor. Changes are both time-consuming and costly. It is essential, therefore, that the final design of a plastic gear be the result of close study and that the data appearing on the drawing be exact, and so specified that there can be no possibility of misinterpretation.

How plastic gears are designed is fully discussed elsewhere, as is also the writing of specifications, but because the data on gear drawings are so often vague and conflicting, it is felt that some comments on the subject may be helpful. There has been published by the American Gear Manufacturers Association, Standards Department, 1500 King Street, Suite 201, Alexandria, Va. 22314, a series of standards relating to the procedures to be followed in writing gear specifications and inspecting finished gears. These standards are universally accepted and should be rigidly adhered to in specifying plastics gearing. The following is a list of the relevant standards:

1012-F90	Gear Nomenclature, Definitions of Terms with Symbols.
910-C90	Information Sheet – Format for Fine-Pitch Gear Specification Data.
1003-G93	Tooth Proportions for Fine-Pitch Spur and Helical Gears.
2000-A88	Gear Classification and Inspection Handbook.
1006-A97	Tooth Proportions for Plastic Gears.



COVER PLATE WITH 36 D.P. INTERNAL GEAR.

Other publications that will be found useful to the gear designer are:

Buckingham, Earle: "Manual of Gear Design"
Sections 1,2 and 3
Buckingham Assoc., Inc.
Parker Hill Road
Springfield, Vt. 05156

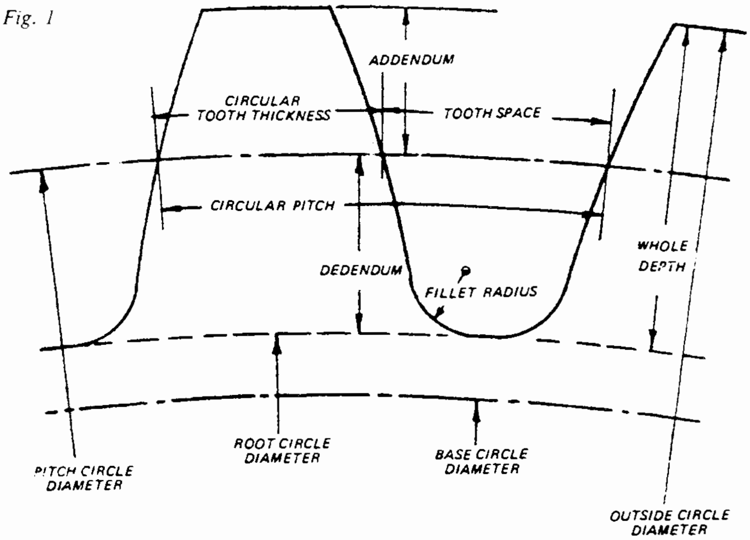
Dennis P. Townsend, Editor: "Dudley's Gear Handbook"
Second Edition
McGraw-Hill, Inc.
New York, NY

Van Keuren Co.: "Van Keuren Precision Measuring Tools Catalog and Handbook"
The Van Keuren Company
Watertown 72, Mass.

ACCURATE MOLDED PLASTIC GEARS

For ease of reference, the tooth proportions of standard spur and helical gears are illustrated and tabulated in Fig. 1.

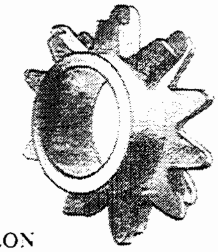
Fig. 1



	Coarse-Pitch 19.99 and Coarser	Fine-Pitch 20.00 and Finer
Addendum	$\frac{1.000}{P}$	$\frac{1.000}{P}$
Dedendum	$\frac{1.250}{P}$	$\frac{1.20}{P} + .002$
Whole Depth	$\frac{2.250}{P}$	$\frac{2.20}{P} + .002$
Circular Tooth Thickness	$\frac{\pi}{2P}$	$\frac{\pi}{2P}$

P = Diametral Pitch

Of all the data to go on the drawing of a gear, none is of greater importance than that defining the tooth thickness required. Since it would appear that there are differences of opinion about just how tooth thickness should be specified, and how a gear should be inspected to insure that the specification has been met, some comments may be in order.



32 D.P. GEAR WITH VARIOUS CAMMING SURFACES: NYLON

ACCURATE MOLDED PLASTIC GEARS

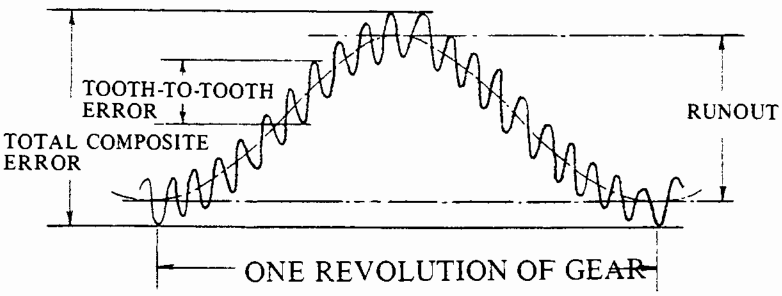
The circular pitch of a gear is the circumference of the standard pitch circle divided by the number of teeth, and the standard circular tooth thickness is half the circular pitch. If two mating gears have standard tooth thicknesses and are brought into close mesh, the distance between their centers will be half the sum of their standard pitch diameters. But two gears having standard tooth thicknesses could operate at the standard center distance only if both gears were perfect. Any errors in the gears would cause them to bind at some point in their rotation, the one with the other.

The errors present in a gear are:

- Runout
- Lateral runout (wobble)
- Pitch error
- Profile error

The pitch error plus the profile error add up to what is called the tooth-to-tooth composite error, and this plus the total runout is known as the total composite error. By rotating a gear in close mesh with a master gear of known accuracy in a variable center distance fixture, the tooth-to-tooth and total composite errors can be determined by noting the radial displacements. If the radial displacements were to be charted, the result would appear as shown in Fig. 2.

Fig. 2



There are center distance measuring instruments available of various types. The simpler models are equipped with a dial indicator and require that the operator note the radial displacements as the gear is rotated manually through 360° in close mesh with the master gear. The more sophisticated models trace the radial displacements, through an electronic device, on a moving chart. Fig. 3 shows one of the simpler models.

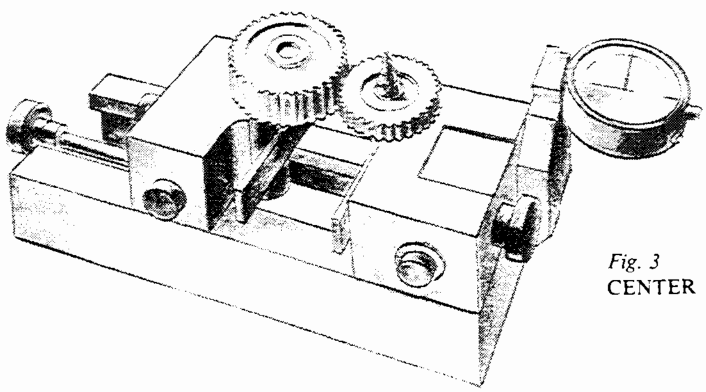
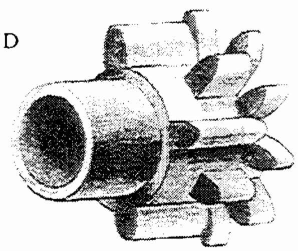


Fig. 3
CENTER DISTANCE MEASURING INSTRUMENT.

ACCURATE MOLDED PLASTIC GEARS

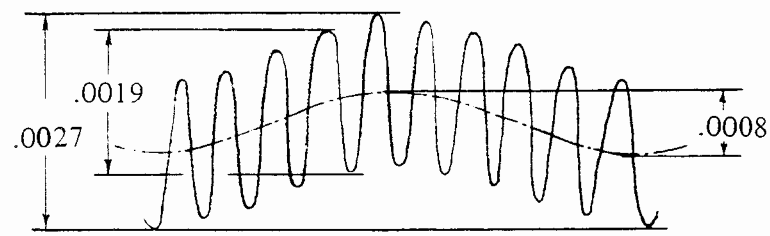
A system has been developed by the American Gear Manufacturers Association whereby gears are classified by number in accordance with their accuracy in terms of the maximum tooth-to-tooth and total composite tolerances allowed them. This number is called the AGMA Quality Number. The AGMA Quality Numbers and the corresponding maximum tolerances, by diametral pitch and pitch diameter, are listed in the American Gear Manufacturers Association "Gear Handbook, 2000-A88."

36 D.P. COMPOUND
PINION: ACETAL



If a gear had assigned to it a Quality Number such that the maximum tooth-to-tooth and total composite tolerances were .0019 and .0027 respectively, and if the errors in the gear were at the maximums allowed by these tolerances, the chart from the center distance measuring instrument would appear as shown in Fig. 4.

Fig. 4



To allow for the errors in two mating gears, either the operating center distance must be made greater than the calculated close mesh center distance by an amount equal to the sum of half the total composite tolerances, or the tooth thicknesses must be thinned by an equivalent amount. AGMA Quality Numbers must be chosen for a pair of mating gears at an early stage in the design procedure, and the finished gears must be inspected by being run in close mesh with a master gear in a center distance measuring instrument to insure that the errors do not exceed the maximums allowed by the tolerances.

Included in the data on the drawing of a gear is the "gear testing radius". The gear testing radius is the center distance between the gear and a master gear, less half the pitch diameter of the master. It has maximum and minimum values corresponding to the maximum and minimum values of the calculated circular tooth thickness, and the maximum total composite tolerance. Just how the testing radius is established can best be explained by working through an example.

A spur gear has 80 teeth, a diametral pitch of 32, and a pressure angle of 20°. The gear is required to have an accuracy corresponding to AGMA Quality Number Q7. The standard pitch diameter is 2.500. From the AGMA "Gear Handbook" it is found that the maximum total composite tolerance is .0036. The gear has a calculated circular tooth thickness of .0460 max. .0445 min. Calculate the testing radius. Assume that the gear will be inspected by being run in close mesh with a master gear having 64 teeth, a pitch diameter of 2.0000, and a circular tooth thickness of .0491.

Calculate the close mesh center distance between gear and master when the circular tooth thickness is at the maximum of .0460 and when it is at the minimum of 0.445.

$$C = \frac{(N_1 + N_2) \times \cos\phi}{2 \times P \times \cos\phi_1}$$

where C = close mesh center distance
N_1 = number of teeth in gear
N_2 = number of teeth in master
P = diametral pitch
ϕ = pressure angle
ϕ_1 = angle whose involute is $\frac{P(t_1 + t_2) - \pi}{N_1 + N_2} + \text{inv}\,\phi$

where t_1 = circular tooth thickness of gear
t_2 = circular tooth thickness of master
π = 3.1415926

$N_1 = 80 \quad N_2 = 64 \quad \phi = 20° \quad P = 32 \quad t_1 = .0460$ max. $.0445$ min. $t_2 = .0491$

$$\text{inv}\,\phi_1 = \frac{32(.0460 + .0491) - 3.1415926}{80 + 64} + .01490438 = .01422110$$

$$\phi_1 = 19.699611°$$

$$\cos\phi_1 = .94147283$$

$$C = \frac{(80 + 64) \times .93969262}{2 \times 32 \times .94147283} = 2.2457$$

and

$$\text{inv}\,\phi_1 = \frac{32(.0445 + .0491) - 3.1415926}{80 + 64} + .01490438 = .01388776$$

$$\phi_1 = 19.549391°$$

$$\cos\phi_1 = .94235339$$

$$C = \frac{(80 + 64) \times .93969262}{2 \times 32 \times .94235339} = 2.2436$$

Close mesh center distance = 2.2457 max. 2.2436 min.

To complete the calculation, half the total composite tolerance is added to the maximum close mesh center distance, half is subtracted from the minimum, and from both results is subtracted half the pitch diameter of the master gear.

$$2.2457 + \frac{.0036}{2} - \frac{2.0000}{2} = 1.2475$$

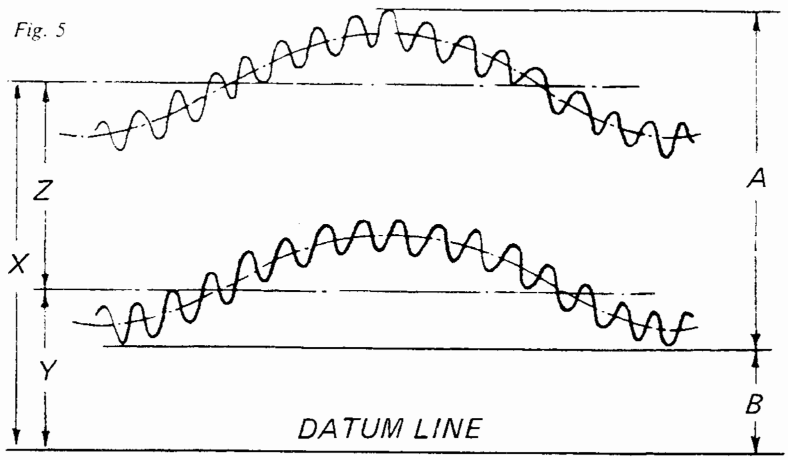
$$2.2436 - \frac{.0036}{2} - \frac{2.0000}{2} = 1.2418$$

Testing radius = 1.2475 max. 1.2418 min.

ACCURATE MOLDED PLASTIC GEARS

If the charts of two samples from the production run of a gear were to be superimposed, the one on the other, they might appear as shown in Fig. 5.

Fig. 5



"*A*" is the difference between the maximum test radius of one sample and the minimum test radius of the other. The tooth thickness of one sample differs from that of the other by an amount equivalent to a radial displacement of "*Z*". If it so happens that the total composite error in each sample is less than what is allowed by the tolerance, then "*Z*" could be greater than the difference between the maximum and minimum calculated tooth thicknesses as specified on the drawing. Yet both samples would be acceptable if their testing radii checked within the maximum and minimum specified values. It is for this reason that the tooth thickness of a gear appears on the drawing as a "basic specification" rather than being included in the "manufacturing and inspection" data, and why it is referred to as the "calculated" circular tooth thickness. It is also the reason why the measurement over pins has a proviso to the effect that this measurement is to be used "for set-up only".

Shown in Figs. 7-1 and 8-3 of "A System for Involute Spur and Helical Gears Molded of the Plastics" are the data that should appear on the drawings of spur and helical gears, in formats recommended by the American Gear Manufacturers Association. Having chosen the number of teeth; the tooth form; the helix angle, if applicable; the AGMA Quality Number; and having determined the tooth thickness; the remaining data are arrived at by mathematical computation: only the tolerance to be given to the outside diameter is a designer's choice.

Specifying the tooth thickness and accuracy of a plastic gear in terms of a testing radius precludes any possibility of misinterpretations and makes inspection a simple and quick operation. If the gear checks within the maximum and minimum values specified for the testing radius, and satisfies the total composite and tooth-to-tooth maximum tolerances, it must, of necessity, be correct in all other respects.

Because bevel gears, hypoid gears and worm gears are commonly manufactured as mating pairs, designing and specifying plastic molded gears in these categories present problems not encountered with spur and helical gears. This is particularly true if the pinion or worm is to be made of metal as is often the case. Then the project requires close co-operation between the molding die maker and the manufacturer of the metal part. For these reasons the authors suggest that the designer should proceed with the preliminary work in accordance with the information contained in the relevant American Gear Manufacturers Association standards and then consult with the molder, the molding die maker and, where applicable, the manufacturer of the metal pinion or worm before completing the design.

ACCURATE MOLDED PLASTIC GEARS

HOW ACCURATE ARE MOLDED PLASTIC GEARS? WHAT AFFECT DOES MOLD SHRINKAGE HAVE ON ACCURACY?

It has frequently been said that molded gears are not as accurate as machined gears. This is a meaningless statement. A molded gear held to the tolerance of AGMA Quality Number Q8 cannot be other than just as accurate as a machined gear of the same quality number. It is true that gears have not yet been molded to the highest precision obtainable by machining, but the number of gears requiring such precision represents only a very small percentage of all the gears made. Furthermore, accuracy is improving with every gear molded. Fine-pitch instrument gears are now being molded to tolerances that would have been considered impossible of achievement in the recent past. As an example, a four cavity molding die for a fine pitch spur gear in acetal produced gears that, on first sampling, were within the tolerances of AGMA Quality No. Q12. They had a tooth-to-tooth error of .0003, or less, and a total composite error of .0005 or less.

It has also been stated that molded plastic gears need not be as accurate as metal gears. This idea is based upon the contention that, because of the yielding nature of plastic, runout and tooth-to-tooth errors do not have the same ill effects. This is quite wrong. Plastic gear teeth that are flexing to an excessive degree because of inaccuracies will fail through fatigue and wear much earlier than accurate teeth.

All plastics shrink in changing from the liquid to the solid state, and as they cool down to room temperature. As a consequence, all mold cavities must be made larger than the parts molded in them. For example, if a molded gear is to have an outside diameter of 1.200 ins. and the plastic has a mold shrinkage of .025 ins. per ins. the outside diameter of the cavity will require to be 1.230 ins.

In making a gear cavity, however, it is not sufficient to take the same generating hob that would be used to cut the teeth in the gear if it were to be machined, and with that hob cut an oversized gear which, in turn, would be used to form the cavity. This is a common mistake that results in a molded gear with a serious profile error. It would have a tooth-to-tooth error larger than anything acceptable.

Fig. 6 is an enlargement of a 32 D.P., 20° P.A., gear tooth and, superimposed upon it the profile of an oversize gear tooth cut with a standard 32 D.P., 20° P.A. hob.

Fig. 7 again shows the standard gear tooth and this time superimposed upon it, the profile of the molded tooth (after shrinkage) that would be obtained from the oversize cavity.

Fig. 6

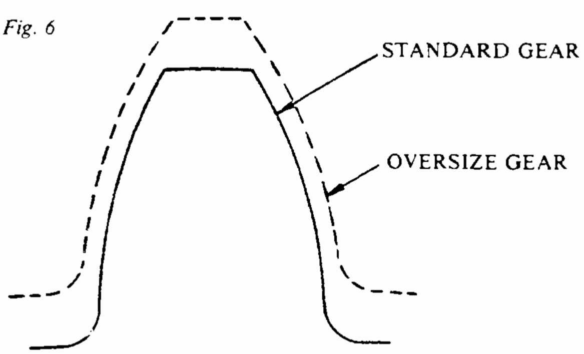
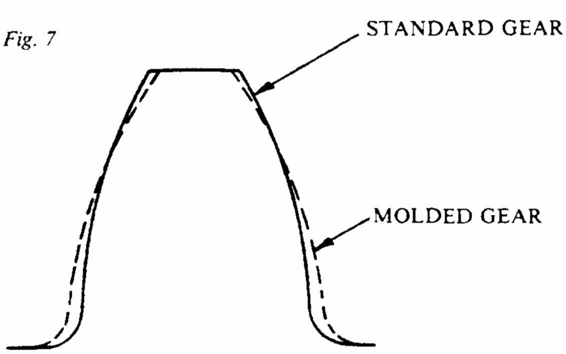


Fig. 7



It will be noted that the tooth of the molded gear departs considerably from standard. It is thicker at the root and thin at the tip: it has a pressure angle much in excess of 20°.

ACCURATE MOLDED PLASTIC GEARS

The amount of pressure angle error can be calculated from:

$$\cos \phi_2 = \frac{D \cos \phi_1}{D(1 + S)}$$

where:

D = Pitch circle diameter
ϕ_1 = Pressure angle of hob
ϕ_2 = Pressure angle of molded gear
S = Shrinkage

If, for example, the pitch circle diameter is 1.000 ins., the pressure angle of the hob is 20° and the shrinkage .025 ins. per in., the pressure angle of the molded gear would be:

$$\cos \phi_2 = \frac{1.000 \times .93969262}{1.000 \times 1.025}$$

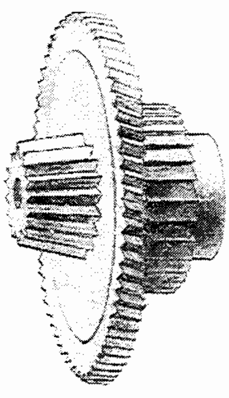
$$= .91677329$$

$$\phi_2 = 23° 32' 28''$$

This amount of error would result in binding, rapid wear and general mal-functioning.

It will be readily apparent that the teeth in the cavity must be carefully compensated for shrinkage so that, when the molded gear solidifies and becomes stable, the teeth will have the correct profile. This design work is further complicated in the case of a helical gear, as the axial shrinkage is usually quite different from that obtaining across the diameter. Compensating correctly for shrinkage in a gear requires of the mold designer a thorough understanding of gear geometry plus considerable experience in the shrinkage behavior of all types and grades of plastics.

The importance of correctly compensating for shrinkage cannot be over-emphasized. If reference is again made to Fig. 4 it will be seen that this gear can have a total composite error of .0027 and a tooth-to-tooth error of .0019. If the tooth-to-tooth error is up to the maximum, the runout must be held to .0008 ins. T.I.R. But if the tooth-to-tooth error is reduced to .0005 — an amount quite possible of achievement if the cavity is correctly designed and accurately made — it will be seen that the runout can go as high as .0022 ins. T.I.R. Since it is more difficult to control runout than tooth-to-tooth error, as will be discussed in Chapter 7, it is of importance that the profile of a plastic molded gear be as accurate as possible — several degrees more accurate than a comparable machined metal gear.



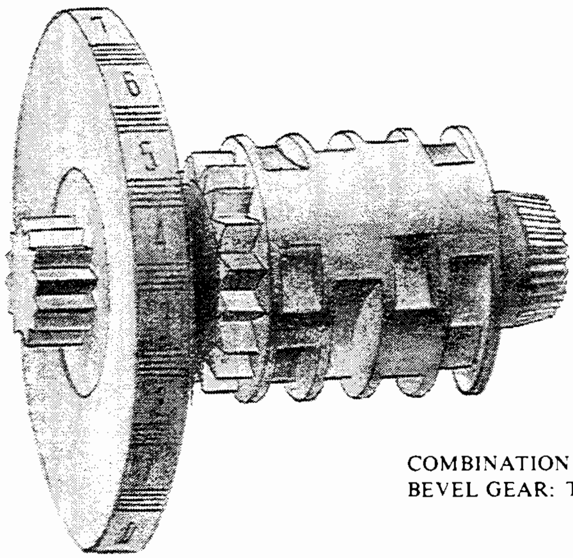
28 D.P. BEVEL PINION, 64 N.D.P. HELICAL GEAR AND RATCHET: MOLYBDENUM DISULPHIDE - FILLED NYLON

ACCURATE MOLDED PLASTIC GEARS

Shrinkage need have no effect on accuracy other than the very minor variations in shrinkrage that occur during the course of a production run, and are allowed for in the tolerances given to the molded gear. These minor variations are due to such factors as slight deviations from standard of the molecular weight of the plastic throughout a batch and changes in operating temperature, but a good gear molder can control these variations within very close limits and so hold all the gears in a lot to print tolerances.

It is not an uncommon practice for designers to specify close tolerances for the outside diameters of plastic gears and leave everything else wide open. This is probably done in the mistaken belief that the outside diameter of a molded gear is a measure of overall accuracy, and because this is the easiest dimension to measure. In fact, all that this close outside diameter tolerance insures is that all the errors present in a lot of molded gears will be of the same magnitude from piece to piece. Except in very rare instances, the outside diameter of a gear is, within limits, a matter of no consequence. If it is specified that the tooth thickness of a molded gear is to be held to +.000 −.001, then the outside diameter must, of necessity, be permitted to vary within a tolerance band of at least .0027 in the case of 20° pressure angle gears and .0039 in the case of 14½° pressure angle gears. For gears given more tooth thickness tolerance, the outside diameter tolerance bands would be greater in the same proportion.

To specify close tolerances for the outside diameter of a molded gear except in the rare cases where the outside diameter is functional — as in pump gears, for instance — can make for unnecessarily high tooling costs and piece prices without in any way guaranteeing that the gears will be accurate in other respects. It cannot be emphasized strongly enough, or often enough, that the accuracy of plastic gears should be specified in terms of AGMA Quality Numbers and should be inspected by the center distance measuring method. No other procedure that is at all practical will insure that the gears will have the desired accuracy.



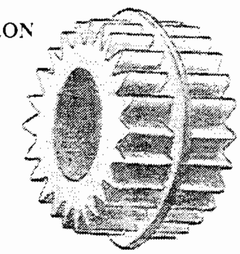
COMBINATION PINION, THUMB WHEEL, RATCHET, CAMS AND BEVEL GEAR: TFE AND GLASS-FILLED POLYCARBONATE.

WHO ARE THE GEAR MOLDERS?

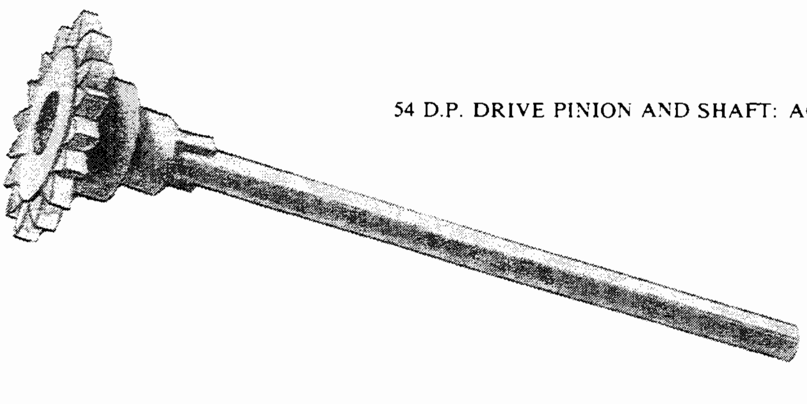
The selection of a molder with whom to place an order for gears should be given careful consideration. Many a purchaser of molded plastic gears has gone through the frustrating experience of waiting weeks for production samples only to find these samples quite unacceptable. There have followed more weeks of waiting while the molder attempts to make corrections, often without much success. The purchaser is then faced with the alternative either to accept poor quality gears, against his better judgment, or to find another molder, with no assurance of any better results.

While it is true that, given a correctly designed and accurately made molding die, any competent molder can produce gears, there are few molding concerns that can be classified as gear manufacturers. A gear molder, however, can be distinquished from a molder who will mold gears; his experience in molding a wide range of gears will qualify him to advise on product design and the selection of materials; he has the molding presses most suitable for gear molding; and he has the inspection equipment necessary to maintain proper quality control.

36 D.P. DOUBLE PINION AND SHROUD: NYLON



The experienced gear molder will not accept an order if he is unsure of meeting print specifications. In such an event, a meeting of the purchaser's engineers and the molder and his molding die maker will usually produce acceptable modifications that will insure the gear being molded within tolerance. In fact, such a meeting is desirable whether or not any problems are anticipated. A full discussion of a molded gear project by the three parties directly concerned will have the end result of making certain that the gear will be right for the application under review, that the data will be correctly interpreted, and that the method of inspection to be followed is agreed upon and fully understood by the molder and his customer's inspection department.



54 D.P. DRIVE PINION AND SHAFT: ACETAL

One yard-stick by which to measure the capabilities of a gear molder is inspection equipment. A molding company that does not have the necessary equipment with which to check gears to insure that they are within the specified tolerances is unlikely to prove a satisfactory vendor.

A gear molder does not need to be a gear engineer. He must, of course, understand how to interpret gear data well enough to know what is required in molding. The knowledge of gearing, and the ability to translate that knowledge into accurate gear molding dies with correct cavities, rests with the gear molding die maker.

ACCURATE MOLDED PLASTIC GEARS

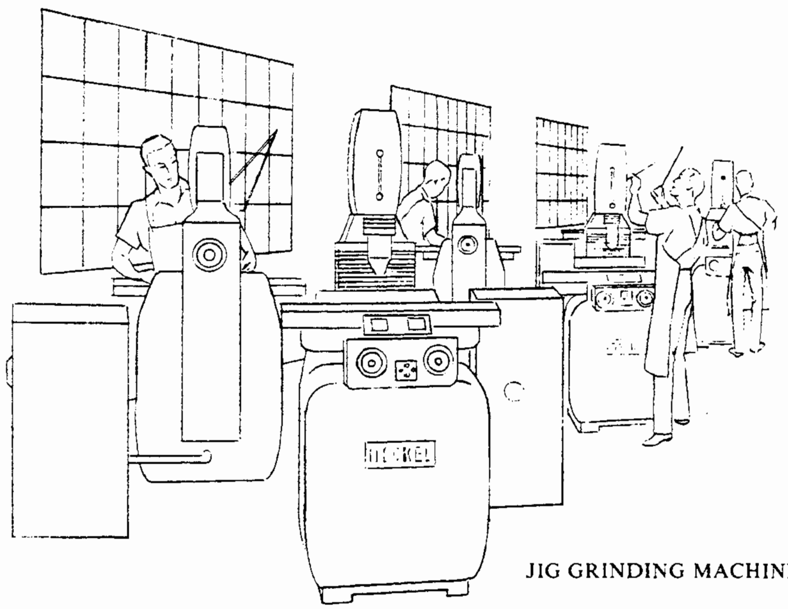
HOW ARE ACCURATE GEAR MOLDING DIES MADE?

A molded gear, of course, can be no more accurate than the molding die which produces it. Subtract the tolerances required for molding from the accuracy of the die and the result is the accuracy that can be expected in the gear.

The first requirement, as pointed out in chapter 6, is a complete understanding between the gear user, the molder and the molding die maker. This is the only way to insure that realistic tolerances are specified, that the product design is adequate for its purpose and that it is suitable for molding, that there is a clearly defined inspection procedure, and that the die maker understands all of this so that he can design and construct a molding die compatible to the requirements.

Basic considerations for gear molding dies are the same as for any accurate product. Molded gears being usually small in relation to the size of average moldings, lend themselves admirably to being molded in the small high speed automatic molding presses developed in recent years. This allows for compact dies with a limited number of cavities.

There are several advantages to this size range of molding die work. The most important advantage is that the construction of completely hardened or case hardened die frames is economically feasible. With the considerable increase in frame strength and the limited press clamping tonnages required for their size, these frames are considerably better able to withstand the abuses of the molding process and can maintain their accuracy throughout extended useful life. Of course, this assumes that the accuracy will have been built in in the first place. With adequate jig grinding equipment this can be readily accomplished. With true alignment of cavity bores, guide pin bores, ejector pin bores, tolerances can be held much closer, enabling minimum running fits to assure flash free moldings and minimum wear. The elimination of wear bushings allowed by hardened steel surfaces in the bores removes the additional errors resulting from less than perfect concentricity of these items. The compact arrangement of cavities limits cavity misalignment because of uneven thermal expansion of the die halves that can be caused by uneven temperatures in these halves.



JIG GRINDING MACHINES

ACCURATE MOLDED PLASTIC GEARS

Other basic considerations are also valid. There must be adequate channeling for accurate control of die temperatures. There must be properly designed and sized runner and gating systems made as accurately as the cavities themselves to obtain the truest flow of plastic material to all sections of all the cavities at as even a pressure as is possible. There must be adequate venting in the proper places to allow air to be readily displaced by the flow of plastic again to insure even flow of the melt. There must be ample ejection systems to insure minimum distortion of the product on its ejection from the die. There must be adequate interlocks between the die halves to remove the misalignment of running fits provided in the guide system. These features must not only be designed in by competent design engineers, but this design must be faithfully followed by experienced die makers with precise equipment and ample inspection devices. A schematic cross section of such a gear molding die is illustrated in Fig. 8.

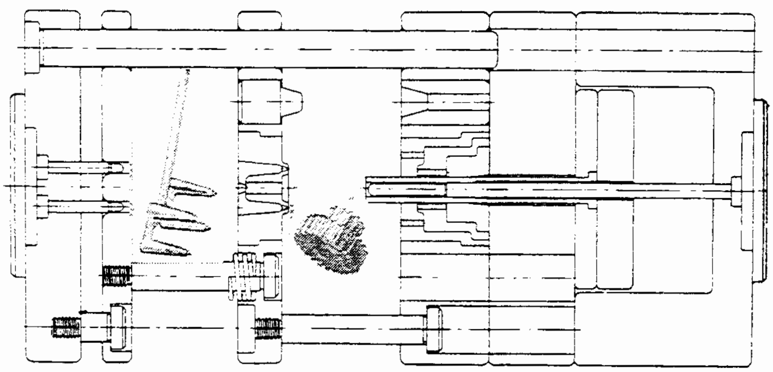


Fig. 8 DIE OPEN

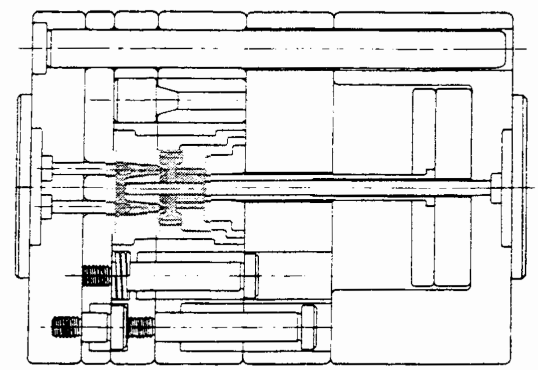


Fig. 8 DIE CLOSED

Beyond basic considerations, however, there are specific problems in gear molding. These can be broken down into the basic essentials of the gear itself. These are tooth-to-tooth error and total composite error. Curiously enough, tooth-to-tooth error, as will be explained later, is not the problem that total composite error is to the molding die maker.

Remembering that total composite error is the sum of tooth-to-tooth error and runout, the problem is simply runout. Starting with the molded gear bore which must be accurate in size and is usually longer than its diameter to be a proper mounting for the gear, creates the first problem. The core pin which molds such a bore must be anchored in both sides of the die, first to insure that any incipient flash will not encroach upon the tolerance of the bore and second to be rigid enough to resist molding pressures. While this pin is closely fitted into the moving half of the die it must be allowed a running fit into the stationary half. The closest of running fits will allow the possibility of .0001/.0002 of siding to take place. If sleeve ejection is required around this core-pin, the solid anchor is lost and an additional running fit will allow an additional .0001/.0002 of potential siding plus whatever eccentricity there is between the inside and outside diameter of the sleeve. Add to these the probability that the gear cavities themselves are a series of concentric rings, especially in cluster gearing, and it becomes apparent that almost perfection in die making still leaves potentials for runout error. Add to that the molding problem that this gear bore creates by eliminating the most desirable gating position in the true center of the gear. This insures that there must be some unevenness of plastic flow, regardless of how many gates are used, that will result in something less than a perfectly round gear. Therefore, more runout. The magnitude of runout errors is the limiting factor in the accuracy of molding gears of today.

Tooth-to-tooth error, the shoal on which many a well-intentioned but uninformed molding die maker has foundered, can be virtually eliminated. At the very least, it can be held well within the tolerances of most plastic gear applications of today. Therefore, the widely held belief that an accurate gear molding die is obtained only after a number of trial cavities have been made is simply not valid. Competent gear molders are able to predict molding shrinkage within close limits, especially with the filled plastics so often specified. Competent gear molding die makers are able to translate this information into correctly compensated gear cavity dimensions, and then to hold these dimensions in the manufacturing process. The days of "cut and try" are over.

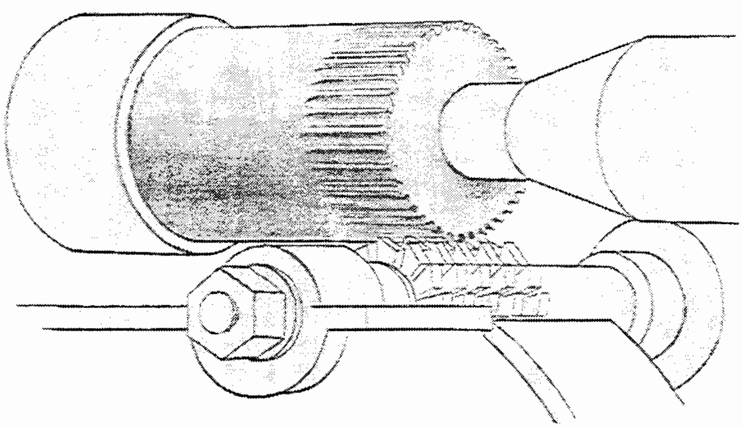
The procedure starts with the examination of the data specified for the molded gear. Once this is determined to be explicit and non-conflicting, shrinkage allowances can be calculated to arrive at the exact dimensions that the cavity will require. Inspection reference dimensions can also be calculated and projection charts can be produced to insure that these dimensions will be held.

The actual manufacture of accurate gear cavities is usually accomplished by ram electrical discharge machining (EDM); whether the configuration is spur, helical, worm, bevel, or hypoid; whether or not the gear and the cavity design allows for a thru-burn (no bottom to the cavity section allowing the electrode to continue to pass on through) or a stop burn; whether the bottom of the cavity is flat or has configuration such as in a mutilated pinion. This discharge machining is done in previously hardened blanks to surface finishes that are fine enough to act as the final finish of the molding cavity. With no further processing to be done, the integrity of the discharge machined gear teeth remains intact. There is no tooth-to-tooth error introduced after discharge machining.

The possible sources of error for ram EDM are reduced then to the manufacture of the electrode for discharging and the discharge process itself. (The final grinding of the cavity blank introduces no error.) Today's electrical discharge machines, with proper fixturing and control devices, in the hands of highly skilled operators who not only understand their process but who also know what is required in a gear cavity and who know the die making procedures as well, can truly translate the electrode forms into a finished gear tooth form. Even with this skill, however, it is impractical in discharge machining to hold sufficient concentricity of the gear form to the blank itself. Stock is therefore allowed on the discharge blank to allow accurate grinding of the finished cavity to hold as close a concentricity as possible of the gear form to the remainder of the molding die components. This, of course, is to hold the total composite error, or runout, to a minimum.

ACCURATE MOLDED PLASTIC GEARS

The procedure between the design of the gear cavity and ram electrical discharge machining is the design and manufacture of the electrode. It has been shown that none of the other processes introduce any appreciable tooth-to-tooth error. Therefore, if this electrode is accurate, tooth-to-tooth error, as stated, can be held to reasonable minimums.



GENERATION OF A GEAR ELECTRODE.

The design of the electrode is identical to the design of the finished gear tooth form blank with one exception. That exception is the spark gap allowance for the discharge machining process. Experienced molding die makers can predict this allowance closely for their particular equipment and methods. The gear molding die maker, knowing that the smaller this gap is, the better control he has in accuracy and surface finish, even though the machining is considerably slower, works to minimum spark gap allowances.

Accurate gear electrodes are generated in a manner similar to the manufacture of gears of the same configuration on similar types of equipment. The difference between the processes is only in the accuracy required. Gear molding die makers, since one electrode may in turn provide the cavity that will produce millions of gears, take the utmost of care in this process to insure that the equipment is in excellent condition and that the operation of it is entrusted to only highly skilled die makers. Since the nature of the generating operation itself is self-cancelling as far as spacing errors in the teeth are concerned, and, because extreme care can be and is taken to assure that the form and size is correct, this final source of error is reduced to acceptable minimums.

Wire electrical discharge machining is an alternate to ram but only for spur gearing and only for those cavity configurations that allow for a thru-burn. Properly done, wire can create cavities with all of the features and be equal in quality to those produced by ram. Unfortunately, there are too many wire software gear design programs in use that improperly develop the trochoid area (root below the involute) and some that only crudely approximate the working involute area of the gears. Software such as this can only lead to gears of less strength, less efficiency, and, in some instances, to interference between mating gears.

The equipment and the skills are available now to manufacture truly accurate gear molding dies. The cost of the extra care required to do so is relatively small in proportion to the cost of ordinary gear molds. This extra cost, when amortized over the production of thousands of gears, is hardly noticeable. It remains only for the purchaser to decide that this is what he wants.

A SYSTEM FOR

INVOLUTE SPUR AND HELICAL GEARS

MOLDED OF THE PLASTICS

BY
WILLIAM McKINLAY

REVISED EDITION

ABA-PGT INC

INTRODUCTION

The gear tooth is the result of centuries of evolution. The techniques employed to form teeth in gears have improved over the years; an entire system of mathematics has grown up around the design of gearing; many text books on the subject have been published; yet the involute gear of today differs little from what scientists and mathematicians conceived some three or four hundred years ago. No pretense is made that "A System for Involute Spur and Helical Gears Molded of the Plastics" is anything other than variations on an old theme; but variations attuned to the plastics as gear materials.

The system has been prepared for engineers having no more than a general knowledge of gearing. It is sufficiently comprehensive in scope that its use requires reference only to two other sources of information: the American Gear Manufacturers Association "Gear Handbook, 390.03", and a set of mathematical tables. For ease of communication, the system is referred to in the text as the "PGT System", and the tooth forms employed as the "PGT Tooth Forms."

INTRODUCTION TO THE REVISED EDITION

There are two approaches to section B. For lay people, the text provides an overview of the particulars that plastics gear engineers consider in their search for a proper design. It spells out the basic reasons why good plastics design departs from the standards for metal gearing. They may skip the examples. For engineers, it lays the groundwork for the actual determination and specification of their application. And, the accompanying PGT software gives both novice and experienced designers the tools they need to accomplish their mission. Appreciation is due to Donald S. Ellis, Raymont M. Paquet and John C. Stone of the ABA-PGT staff who have updated some previous equations and have developed the computer software that enhances the usefulness of this work.

Samuel D. Pierson

Manchester, Connecticut 1994

SPUR & HELICAL GEARS

CONTENTS

SECTION B

SPUR & HELICAL GEARS

SPUR & HELICAL GEARS

THE INVOLUTE CURVE

To employ intelligently any gear system based upon the use of the involute tooth form, it is desirable to have an understanding of the geometry of the involute curve. The classical explanation of this curve requires one to imagine a length of string wound around a circular disc, and then unwound while the string is kept taut. The end of the string will describe, in space, an involute curve. Fig. 1-1 shows the string unwound through an angle β. The disc is called the base circle.

Fig. 1-1

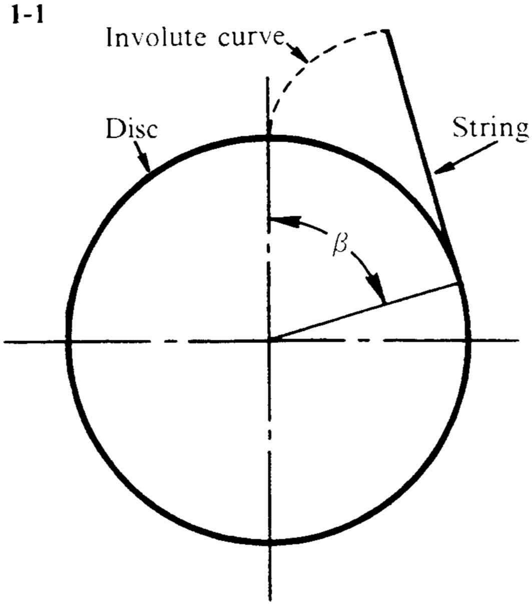
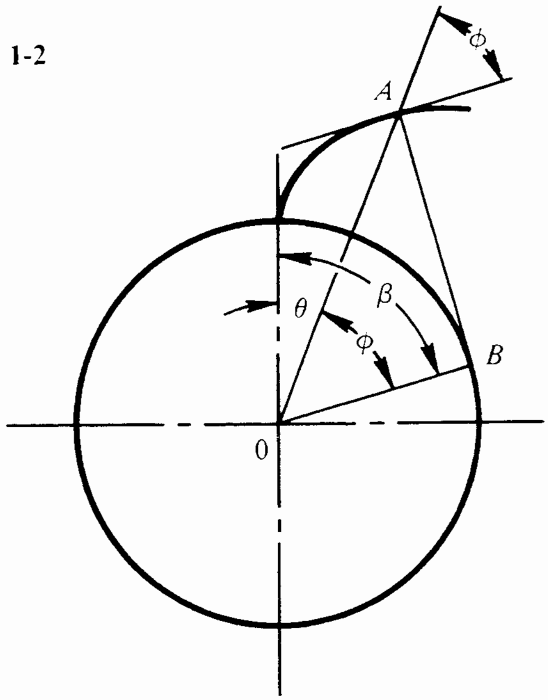


Fig. 1-2



Consider now Fig. 1-2

A is any point on an involute curve. The line AB is equal to the arc subtended by the angle β.

$$AB = OB \times \beta \text{ radians}$$

$$\beta \text{ radians} = \frac{AB}{OB} = \tan \phi$$

$$\theta = \beta - \phi$$

$$\theta \text{ radians} = \tan \phi - \phi \text{ radians}$$

$\tan \phi - \phi$ radians enters into many gearing calculations. To make these calculations less cumbersome, the value of $\tan \phi - \phi$ radians is called the involute of ϕ and is written inv ϕ.

Example 1-1

Find the involute of 20°

tan 20° = .36397023   20° = .34906585 radians

inv 20° = .36397023 − .34906585

= .01490438

Further to simplify gearing calculations, tables of involute functions have been compiled. Perhaps the best known of these tables are those contained in section one of the "Manual of Gear Design" by Earle Buckingham. Incidentally, all three sections of this manual should be in the library of every gear designer.

Computer programs are now available to greatly simplify finding an inverse involute. For example, see "Dudley's Gear Handbook, Second Edition," pp. 6.6 - 6.9.

In the language of gearing, the angle ϕ is called the pressure angle, the point A is called the pitch point, and OA, the radius from the center of the base circle to the pitch point, is called the pitch circle radius.

SPUR & HELICAL GEARS

From Fig. 1-2

$$OB = OA \times \cos\phi$$

Equation 1-1

$$D_b = D \times \cos\phi$$

where

D_b = base circle diameter
D = pitch circle diameter
ϕ = pressure angle

Involute gear teeth are cams having the same involute profile and equally spaced around the base circle. When one gear drives another, the cams of the driver act against those of the driven, in succession, one pair of mating cams taking over the drive before the preceding pair part company.

Fig. 1-3

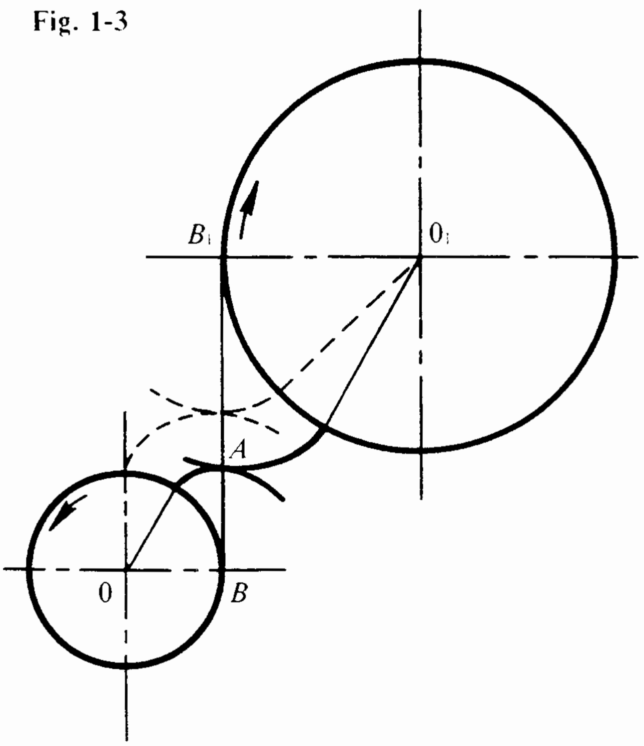
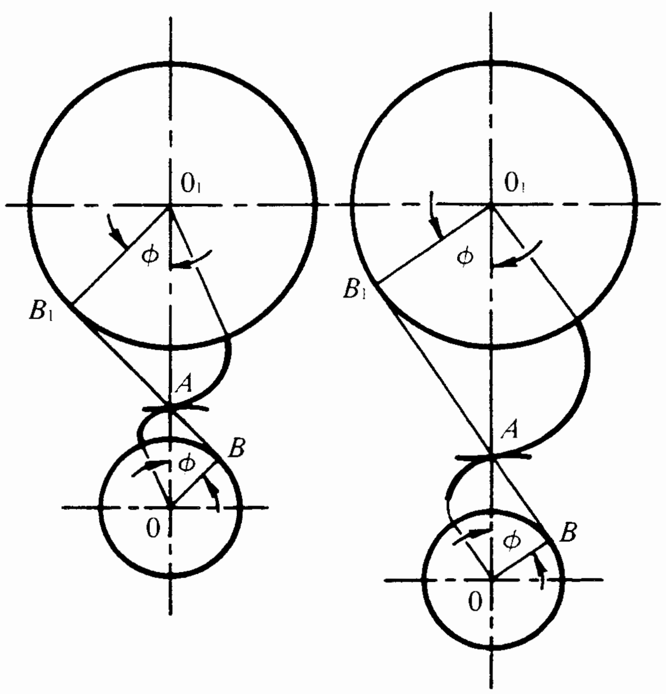


Fig. 1-4



When one involute cam acts against another, as shown in Fig. 1-3, the locus of the points of contact, BB_1, is the common tangent to the two base circles and is called the line of action. If the smaller of the base circles were to rotate at a uniform rate of speed in the direction indicated, the line BA would lengthen uniformally. At the same time, the line AB_1 would shorten at the same uniform rate, and the larger base circle would also rotate at a uniform rate of speed. This ability of involute cams to transmit uniform motion makes the involute tooth form preferable to all others for the vast majority of gear drives.

If the larger of the two base circles had twice the diameter of the smaller, the involute of the larger would turn through half the angle of the involute of the smaller. Thus, the base circles would have the same uniform circumferential speed, and the larger would rotate through one half revolution for each revolution of the smaller. Hence the revolutions per unit of time of two mating involute gears are in inverse proportion to the diameters of their base circles and, consequently, to the number of teeth in the gears.

Fig. 1-4 shows the same two involutes in contact, but in one instance the distance between the centers of the base circles is greater than in the other. It is clear that the greater the center distance, the greater is the pressure angle, ϕ, and the greater are the radii of the pitch circles. Thus the pressure angle and pitch circle of a gear cannot be determined until that gear is in contact with another gear at a definite center distance. Furthermore, a pair of mating involute gears will function at widely varying center distances, a feature of the involute tooth form that gives the designer considerable flexibility in arriving at the best possible arrangement for the drive under review.

On every drawing of a gear are specified the pressure angle and the standard pitch diameter. This simply means that at that particular diameter the pressure angle is that specified. The actual, or operating pressure angle, and operating pitch diameter are not established until the center distance between the gear and its mate has been fixed. The operating pressure angle and operating pitch diameters will be the same as those specified on the drawings only if it so happens that the center distance fixed upon is half the sum of these pitch diameters. This center distance is referred to as the standard center distance. It might appear to make more sense to specify, on the drawing of a gear, the base circle diameter, which has a constant value regardless of the center distance; but long standing convention rules otherwise.

From Fig. 1-4 it will be seen that the center distance of two mating gears is half the sum of their base circle diameters divided by the cosine of the pressure angle.

$$C = \frac{D_{b1} + D_{b2}}{2 \times \cos \phi}$$

and

$$\phi = \cos^{-1}\left[\frac{D_{b1} + D_{b2}}{2 \times C}\right]$$

where

ϕ = pressure angle
D_{b1} = base circle diameter of 1st gear
D_{b2} = base circle diameter of 2nd gear
C = center distance

However, if the standard pitch diameters and the pressure angle at these diameters are known, the equation can be written:

Equation 1-2

$$\phi_1 = \cos^{-1}\left[\frac{(D_1 + D_2) \times \cos \phi}{2 \times C_1}\right]$$

where

ϕ_1 = operating pressure angle
D_1 = standard pitch diameter of 1st gear
D_2 = standard pitch diameter of 2nd gear
ϕ = pressure angle at D_1 and D_2
C_1 = operating center distance

Example 1-2

Given that two gears having standard pitch diameters of 1.000 and 2.000, and a pressure angle of 20° at these diameters, mate at an operating center distance of 1.525, find the operating pressure angle.

$$\phi_1 = \cos^{-1}\left[\frac{(D_1 + D_2) \times \cos\phi}{2 \times C_1}\right] \qquad \text{Equ. 1-2}$$

$D_1 = 1.000 \quad D_2 = 2.000 \quad \phi = 20° \quad C_1 = 1.525$

$$\phi_1 = \cos^{-1}\left[\frac{(1.000 + 2.000) \times \cos 20°}{2 \times 1.525}\right]$$

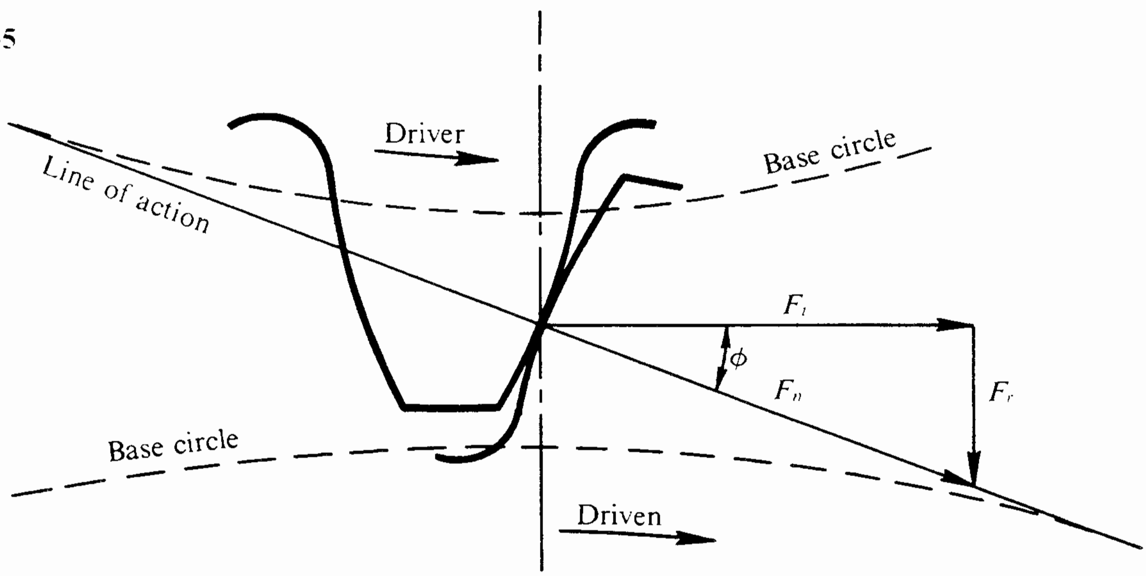
$$= \cos^{-1}\left[\frac{(1.000 + 2.000) \times .93969262}{2 \times 1.525}\right]$$

$$= \cos^{-1}\left[.92428782\right]$$

$$= 22.438791°$$

The gears in this example are referred to as being 20° involute gears. At the standard center distance the pressure angle would indeed be 20°; but, by making the center distance other than standard, the pressure angle can be made what is considered right for the job under review. For example, if an operating pressure angle of, say, 17° was cosidered desirable for these gears, this can be obtained by making the operating center distance 1.474.

Fig. 1-5



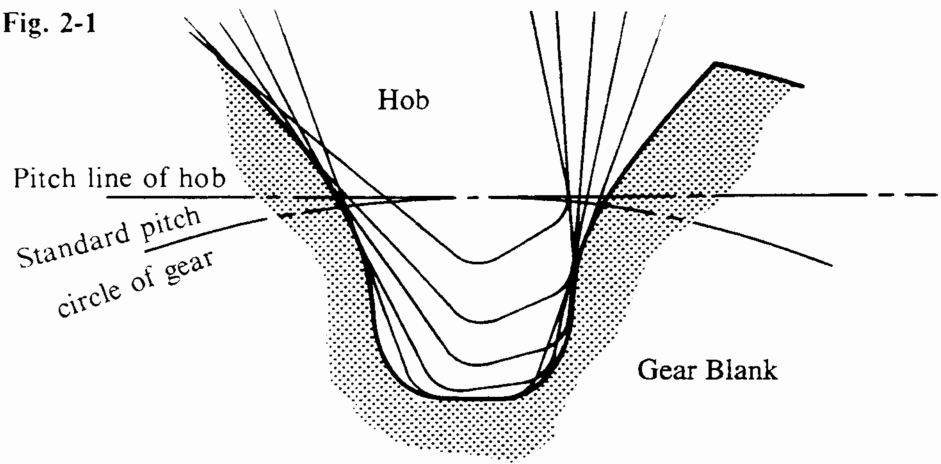
When one involute tooth drives another, as shown in Fig. 1-5, the driving force (F_n) acts along the line of action. The driving force has two components, a tangential component (F_t) and a radial component (F_r). The tangential component represents the useful work done in transmitting the load from the driving to the driven shaft. The radial component represents the work expended in attempting to separate the shafts, i.e., work that is dissipated in the form of heat generated in the bearings.

There are many factors that determine the operating pressure angle of a pair of mating gears, but in applications calling for maximum gear efficiency the operating pressure angle should be kept as low as other considerations will permit.

THE INVOLUTE TOOTH

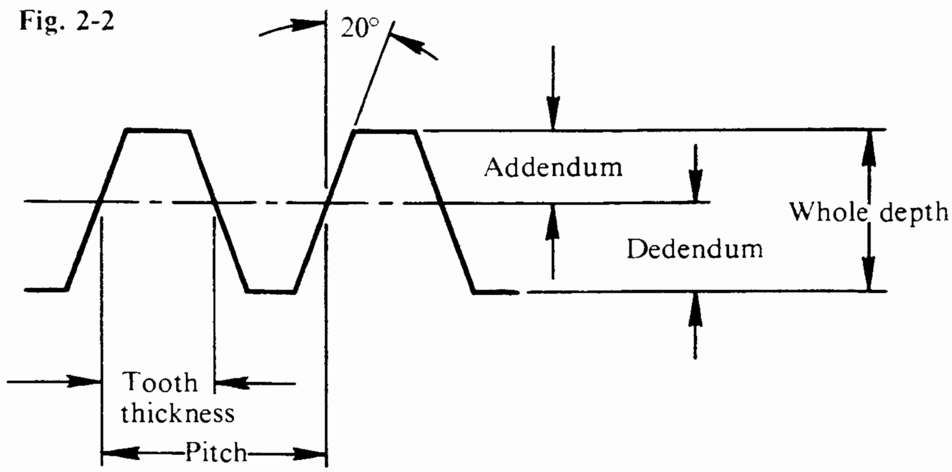
The involute profile can be given to a gear tooth in a variety of ways. The method that best lends itself to explanation is one whereby the teeth are cut by a hob in a gear generating machine. A hob is essentially a fluted worm having straight sided teeth. The hob and gear blank so rotate in the generating machine that the teeth formed as the hob is fed into the blank are given an involute profile, as shown in Fig. 2-1.

Fig. 2-1



The proportions of a gear tooth are specified by reference to the basic rack tooth form. As the name would imply, the basic rack tooth form is the shape of the tooth as it would appear if cut in a rack—a gear having an infinite number of teeth. It would have the same shape as the space between two teeth of the hob. Fig. 2-2 shows the basic rack of a 20° pressure angle gear tooth.

Fig. 2-2



That part of the tooth above the pitch line is called the addendum, and the part below is known as the dedendum. The pitch is the distance between two teeth, and the tooth thickness is half the pitch.

If the hob is fed into the blank until the pitch line of the hob is tangent to the standard pitch circle of the gear, the circular tooth thickness will be half the pitch of the hob and, consequently, half the circular pitch of the gear. The outside diameter of the gear will then be the standard pitch diameter plus two addendums. The root diameter will be the standard pitch diameter minus two dedendums. The pressure angle of the gear at the standard pitch diameter will be half the included angle of the hob teeth. If the rack tooth form were that shown in Fig. 2-2, the pressure angle would be 20°.

SPUR & HELICAL GEARS

Appearing in the basic specifications of a gear is what is known as the diametral pitch. The diametral pitch is a measure employed to specify the size of the teeth, and simply denotes the number of teeth per inch of standard pitch diameter. The larger the numerical value of the diametral pitch, the smaller in size will be the teeth.

Equation 2-1

$$D = \frac{N}{P}$$

where
D = standard pitch diameter
N = number of teeth
P = diametral pitch

Example 2-1

Given that a gear has 64 teeth and a diametral pitch of 32, find the standard pitch diameter.

$$D = \frac{N}{P} \qquad \text{Equ. 2-1}$$

$$N = 64 \qquad P = 32$$

$$D = \frac{64}{32}$$

$$= 2.000$$

In countries employing the metric system, the module is the measure used to specify the size of the teeth in a gear. The metric module is the standard pitch diameter, in millimeters, divided by the number of teeth.

Equation 2-2

$$D = M \times N$$

where
D = standard pitch diameter in millimeters
M = module
N = number of teeth in gear

Example 2-2

Given that a gear has 40 teeth and a module of 1.25, find the standard pitch diameter.

$$D = M \times N \qquad \text{Equ. 2-2}$$

$$M = 1.25 \quad N = 40$$

$$D = 1.25 \times 40$$

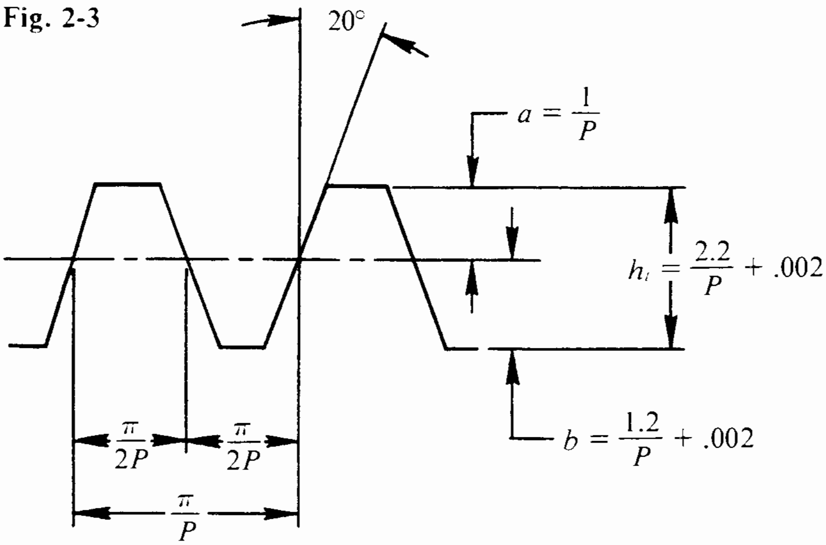
$$= 50 \text{ millimeters}$$

To find the diametral pitch equivalent to a metric module, 25.4 is divided by the module.

$$P = \frac{25.4}{M}$$

In example 2-2 the module is 1.25, therefore the equivalent diametral pitch is 25.4 divided by 1.25, and is 20.32.

Fig. 2-3



Example 2-4

Given that a gear has 32 teeth and a diametral pitch of 64, and that the basic rack tooth form specified is as shown in Fig. 2-3, find the addendum, dedendum, whole depth and standard circular tooth thickness at the standard pitch diameter.

$$\text{Addendum } (a) = \frac{1.0}{P} = \frac{1}{64} = .0156$$

$$\text{Dedendum } (b) = \frac{1.2}{P} + .002 = \frac{1.2}{64} + .002 = .0208$$

$$\text{Whole depth } (h_t) = \frac{2.2}{P} + .002 = \frac{2.2}{64} + .002 = .0364$$

$$\text{Standard circular tooth thickness } (t) = \frac{\pi}{2P} = \frac{3.1415926}{2 \times 64} = .0245$$

The circular pitch of a gear is the standard pitch diameter multiplied by π and divided by the number of teeth.

$$\rho = \frac{D \times \pi}{N}$$

where

ρ = circular pitch
D = standard pitch diameter
N = number of teeth

$$D = \frac{N}{P} \qquad \text{Equ. 2-1}$$

and

$$\rho = \frac{\pi}{P}$$

If the hob is fed into the gear blank so that the pitch line of the hob is tangent to the standard pitch circle of the gear, as shown in Fig. 2-1, the circular thickness of the tooth is half the circular pitch.

Equation 2-3

$$t = \frac{\pi}{2P}$$

where

t = circular tooth thickness
π = 3.1415926
P = diametral pitch

The tooth thickness so obtained can be referred to as the standard circular tooth thickness of a gear having the diametral pitch specified.

Example 2-3

Given that a gear has a diametral pitch of 32, find the standard circular tooth thickness.

$$t = \frac{\pi}{2P} \qquad \text{Equ. 2-3}$$

$\pi = 3.1415926 \qquad P = 32$

$$t = \frac{3.1415926}{2 \times 32}$$

$$= .0491$$

The diametral pitch is also a measure of the other proportions of a tooth—the addendum and dedendum. It is customary to specify the basic rack tooth form in terms of a diametral pitch of 1.0. The actual dimensions are obtained by dividing by the diametral pitch specified for the gear. Fig. 2-3 shows the basic rack tooth form as it might appear on the drawing of a gear. This basic rack tooth form happens to be that of the U.S. standard system for 20° involute fine-pitch gears.

THE PGT TOOTH FORM

The basic rack tooth form of gears in the coarse-pitch category differs from that of fine pitch gears, only to the extent that the dedendum is $1.25/P$ instead of $1.2/P + .002$. Hobs to cut teeth to both standards are available "off the shelf" in all the commonly specified diametral pitches. For economic reasons, designers of commercial cut metal gears will rarely depart from these standard tooth forms. As discussed in "Accurate Molded Plastic Gears", the molding process frees the designer from all such restrictions. He is free to introduce any modifications to the tooth form he may consider beneficial and choose the diametral pitch that is right for the job. These departures from standard add nothing to the cost of a gear molded of the plastics.

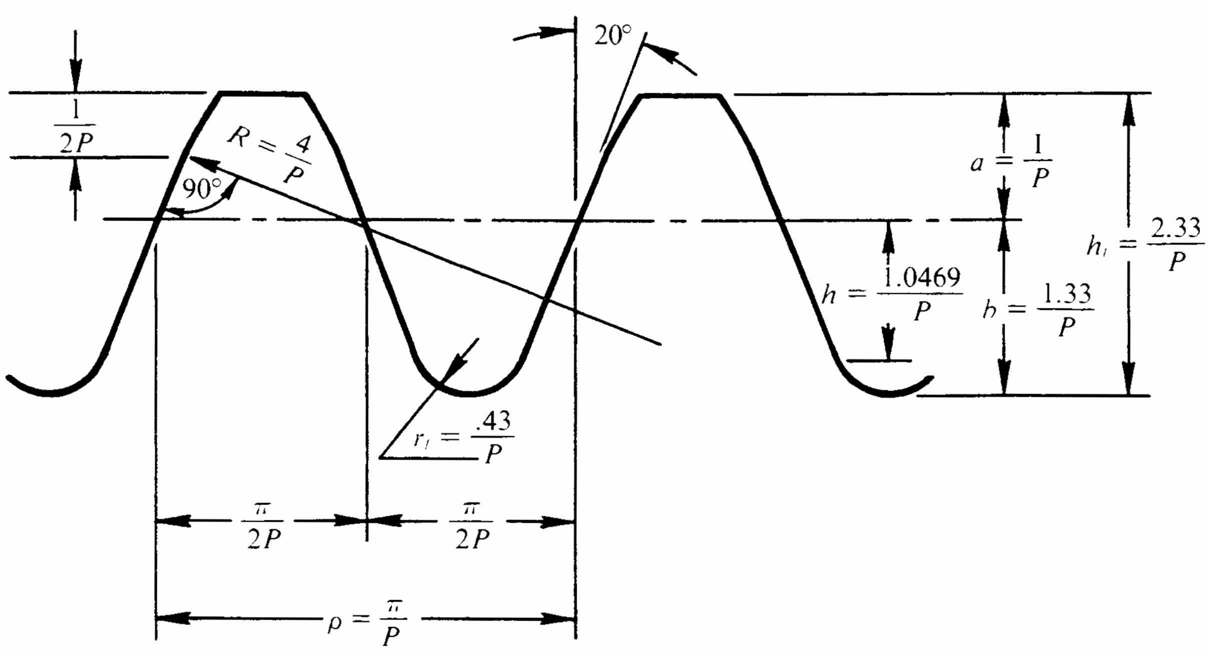
When a tooth deflects under load, the trailing tooth is out of register for truly smooth engagement with the oncoming tooth of the mating gear. The ill effects are considerable—noise, excessive wear and loss of uniform motion. The teeth of heavily loaded metal gears in critical drives are given a degree of tip relief to reduce these ill effects. The teeth are gradually thinned from halfway up the addendum to the tip, providing a "sled runner" effect. Since plastics flex to a greater relative extent than do the metals, the teeth of all plastic gears should have this modification.

The teeth of heavily loaded metal gears are also given a full fillet radius between two teeth at the root. This full fillet root radius can increase the fatigue strength by as much as twenty percent. It is a modification to the standard tooth form that should also be specified for the teeth of plastic gears.

Fig. 3-1 shows the basic rack tooth form of a tooth incorporating these two modifications. It is the basic tooth form of the PGT System. A gear having this tooth form will function perfectly with a mating gear having either the fine-pitch or coarse-pitch standard tooth form.

Fig. 3-1 (PGT-1 TOOTH FORM)

Fig. 3-1



P = diametral pitch ρ = circular pitch 20° = pressure angle
a = addendum b = dedendum h_t = whole depth r_f = root radius
h = depth of straight portion of dedendum to point of tangency with root radius.

SPUR & HELICAL GEARS

Fine-pitch metal gears having teeth conforming to the current standard basic rack tooth form provide a relatively greater amount of clearance between the tips and roots of mating gears than do gears in the coarse-pitch category. The greater clearance is required to allow for the wear of the tips of fine-pitch hobs after prolonged use and repeated grinding. Since hobs are not employed in the manufacture of molded gears, there is no necessity for this additional clearance. The PGT tooth form is specified for either fine-pitch or coarse-pitch gears. The fine-pitch category includes all gears of 20 diametral pitch and up; the coarse-pitch category includes all those having diametral pitches of less than 20.

Fine pitch instrument gears should have teeth that are longer than standard. The relatively large coefficients of linear thermal expansion of the plastics make the use of longer gear teeth mandatory for most plastic gears employed in instrument movements. A pair of mating gears must be so designed that they will not bind at the highest temperature to which they will be subjected. If not provided with longer than standard teeth, at lower temperatures they may well be out of mesh to such an extent that continuity of action is lost.

For fine-pitch gears in electric clocks, control mechanisms, meters, cameras, and similar applications, the PGT System has three additional tooth forms designated PGT-2, PGT-3 and PGT-4. These tooth forms are shown in Figs. 3-2, 3-3 and 3-4. The determination of which tooth form to choose for a specific job will be discussed in a later chapter.

Fig. 3-2 (PGT-2 TOOTH FORM)

Fig. 3-2

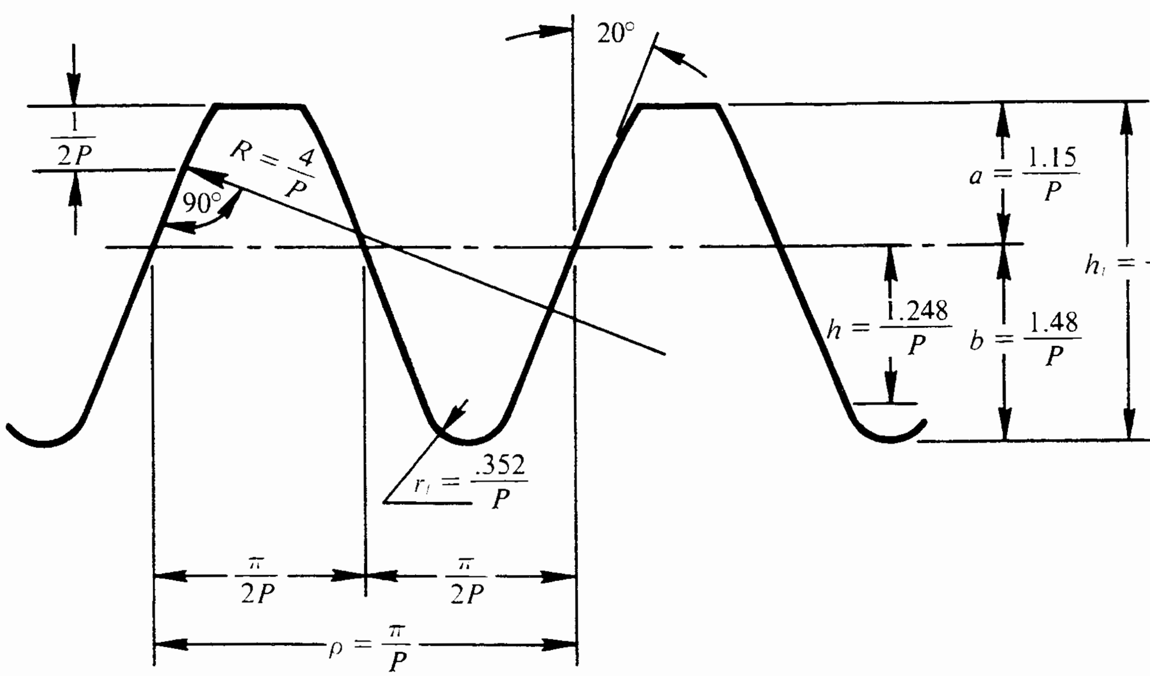


Fig. 3-3 (PGT-3 TOOTH FORM)

Fig. 3-3

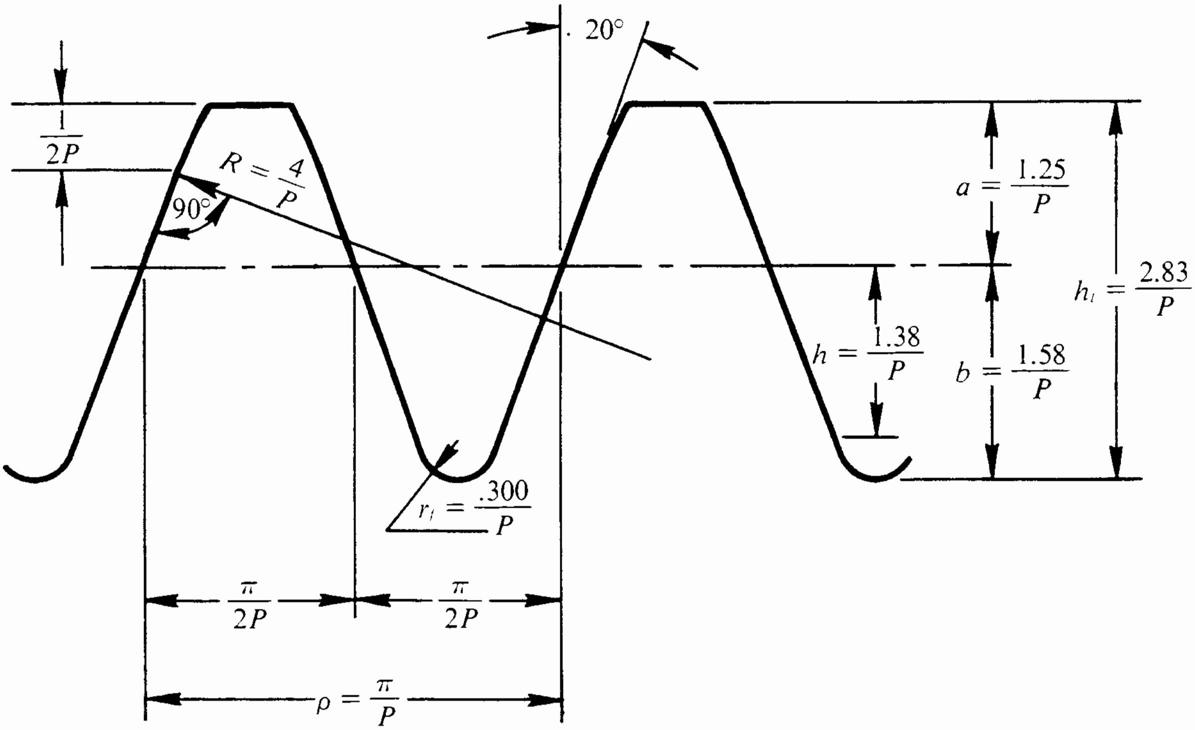
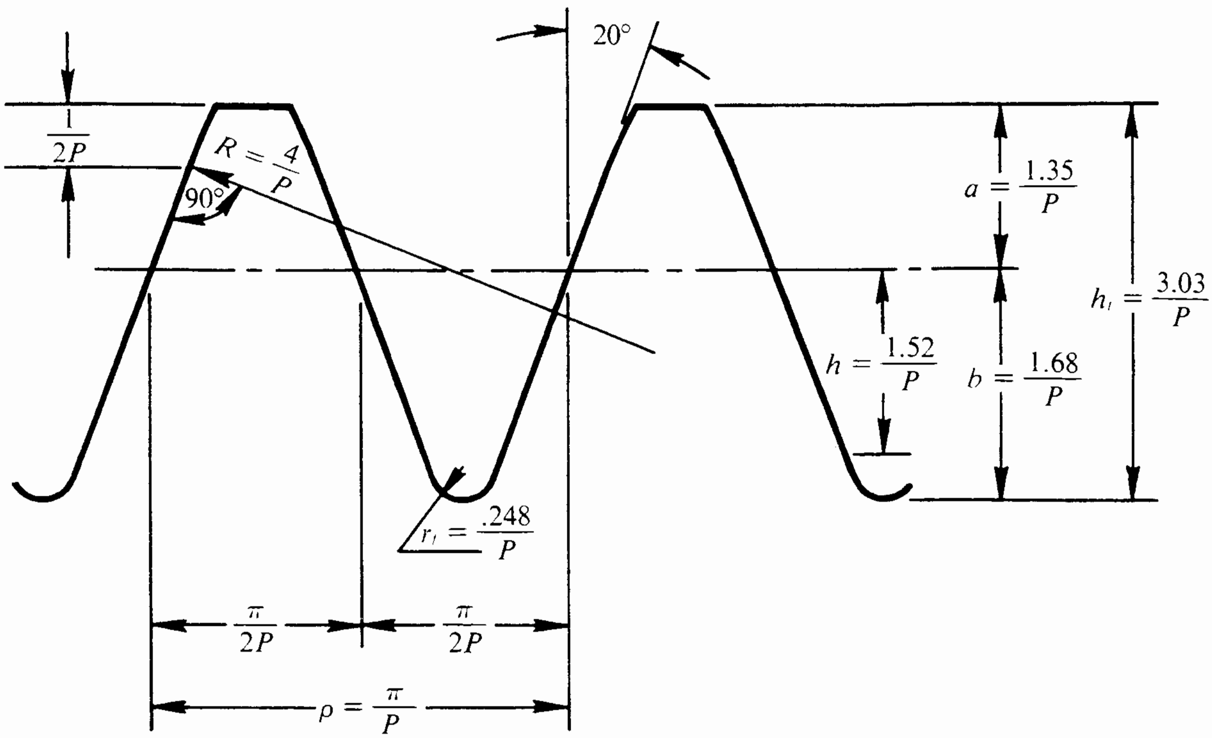


Fig. 3-4 (PGT-4 TOOTH FORM)

Fig. 3-4



TOOTH THICKNESS

The tooth thickness of a gear is always specified as being the circular tooth thickness on the standard pitch circle. If the teeth are generated by a hob, the circular tooth thickness will depend upon the depth to which the hob is fed into the gear blank. In Fig. 2-1 the hob has been fed into the blank until the pitch line of the hob is tangent to the pitch circle of the gear. If the feed of the hob is halted before it reaches that depth, the circular tooth thickness is greater than standard. The increase in thickness is equal to the amount the hob has been held back from the standard depth multiplied by twice the tangent of the pressure angle. The addendum of each tooth is lengthened by the amount the hob has been held back. The so-called long-addendum gears are simply gears having teeth that are thicker than standard. To thin teeth so as to have less than the standard thickness, the hob is fed in until its pitch line is inside the pitch circle. The decrease in tooth thickness is equal to the amount the hob is fed beyond the standard depth multiplied by twice the tangent of the pressure angle.

Thus, there is a direct relationship between the circular tooth thickness specified for a gear and the outside and root diameters. This relationship can best be explained by working through an example.

Example 4-1

A gear has 15 teeth, a diametral pitch of 24 and the PGT-1 tooth form. The circular tooth thickness on the standard pitch circle is .0706. Find the outside and root diameters.

(1) Find the standard pitch circle diameter.

$$D = \frac{N}{P} \qquad \text{Equ. 2-1}$$

$$N = 15 \qquad P = 24$$

$$D = \frac{15}{24} = .6250$$

(2) Find the standard addendum.

$$a = \frac{1}{P} \qquad \text{Fig. 3-1}$$

$$= \frac{1}{24} = .04167$$

(3) Find the standard circular tooth thickness.

$$t = \frac{\pi}{2P} \qquad \text{Equ. 2-3}$$

$$= \frac{3.1415926}{2 \times 24} = .06545$$

(4) The circular tooth thickness specified is .0706. The increase over standard is .0706 − .06545 = .00515. To achieve this increase the hob is held back by .00515 divided by 2 tan ϕ, where ϕ is the pressure angle. The pressure angle of the PGT-1 tooth form is 20° (Fig. 3-1), therefore the hob is held back by

$$\frac{.00515}{2 \times .36397023} = .00707$$

(5) The standard addendum is .04167. The addendum corresponding to the specified tooth thickness of .0706 is the standard addendum plus the amount the hob has been held back.

addendum = .04167 + .00707
= .04874

(6) Find the outside diameter.
The outside diameter of a gear is the standard pitch diameter plus two addendums.

Outside diameter = .625 + (2 × .04874)
= .7225

(7) Find the root diameter.
The root diameter is the outside diameter minus two whole depths. Find the whole depth.

$$h_t = \frac{2.33}{P} = \frac{2.33}{24} = .09708$$

Fig. 3-1

Root diameter = .7225 − (2 × .09708)
= .5283

Generating the teeth by a hob is only one of a number of methods employed to form involute teeth, but the relationship between the circular tooth thickness and the outside and root diameters applies no matter how the teeth are formed.

Given the circular tooth thickness of a gear having one of the four PGT tooth forms, the outside and root diameters are readily obtained by using the appropriate equations in Tables 4-1 and 4-2.

Where:
D_o = outside diameter
D_R = root diameter
P = diametral pitch
N = number of teeth
t = circular tooth thickness on standard pitch circle

Table 4-1

Tooth form	Outside diameter	**Equ. No.**
PGT-1	$D_o = \frac{1}{P}(N - 2.3157) + (2.7475 \times t)$	**4-1**
PGT-2	$D_o = \frac{1}{P}(N - 2.0157) + (2.7475 \times t)$	**4-2**
PGT-3	$D_o = \frac{1}{P}(N - 1.8157) + (2.7475 \times t)$	**4-3**
PGT-4	$D_o = \frac{1}{P}(N - 1.6157) + (2.7475 \times t)$	**4-4**

Table 4-2

Tooth form	Root diameter	Equ. No.
PGT-1	$D_R = \frac{1}{P}(N - 6.9757) + (2.7475 \times t)$	**4-5**
PGT-2	$D_R = \frac{1}{P}(N - 7.2757) + (2.7475 \times t)$	**4-6**
PGT-3	$D_R = \frac{1}{P}(N - 7.4757) + (2.7475 \times t)$	**4-7**
PGT-4	$D_R = \frac{1}{P}(N - 7.6757) + (2.7475 \times t)$	**4-8**

Example 4-2

Find the outside and root diameters of the gear in Example 4-1. This gear has 15 teeth, a diametral pitch of 24, the PGT-1 tooth form and a circular tooth thickness of .0706.

$$D_o = \frac{1}{P}(N - 2.3158) + (2.7475 \times t) \qquad \text{Equ. 4-1}$$

$$P = 24 \quad N = 15 \quad t = .0706$$

$$D_o = \frac{1}{24}(15 - 2.3158) + (2.7475 \times .0706)$$

$$= .7225$$

$$D_R = \frac{1}{P}(N - 6.9758) + (2.7475 \times t) \qquad \text{Equ. 4-5}$$

$$D_R = \frac{1}{24}(15 - 6.9758) + (2.7475 \times .0706)$$

$$= .5283$$

If the two involute curves forming the profile of a tooth are continued out, they will eventually cross and the tooth will be pointed. If a gear has a small number of teeth and an enlarged tooth thickness, the teeth may become pointed at a diameter less than the outside diameter given by the equation in Table 4-1. To insure against the possibility of specifying an outside diameter impossible of attainment, Equation 4-9 is used as a check. The lesser of the answers obtained by use of the equation in Table 4-1 and Equation 4-9 is the outside diameter specified.

Equation 4-9 gives the maximum outside diameter that will still provide the teeth with an adequate top land. It can be used for all four PGT tooth forms.

Equation 4-9

$$D_o\,(\text{max}) = \frac{N \times .93969262}{P \times 1.017 \times \cos\phi_1}$$

where:

D_o (max) = maximum allowable outside diameter
N = number of teeth
P = diametral pitch
ϕ_1 = angle whose involute is $\frac{t \times P}{N} + .01490438$
t = circular tooth thickness

Example 4-3

A gear has 9 teeth, a diametral pitch of 48, the PGT-1 tooth form and a circular tooth thickness of .0406. Find the outside and root diameters.

$$D_o = \frac{1}{P}\,(N - 2.3158) + (2.7475 \times t) \qquad \text{Equ. 4-1}$$

$$P = 48 \quad N = 9 \quad t = .0406$$

$$D_o = \frac{1}{48}\,(9 - 2.3158) + (2.7475 \times .0406)$$

$$= .2508$$

$$D_o\,(\text{max}) = \frac{N \times .93969262}{P \times 1.017 \times \cos\phi_1} \qquad \text{Equ. 4-9}$$

$$\text{inv}\,\phi_1 = \frac{t \times P}{N} + .01490438$$

$$= \frac{.0406 \times 48}{9} + .01490438$$

$$= .2314377$$

$$\phi_1 = 45.933527^\circ$$

$$\cos\phi_1 = .69549246$$

$$D_o\,(\text{max}) = \frac{9 \times .93969262}{48 \times 1.017 \times .69549246}$$

$$= .2491$$

The lesser of the answers given by Equations 4-1 and 4-9 is .2491. It is the outside diameter to be specified.

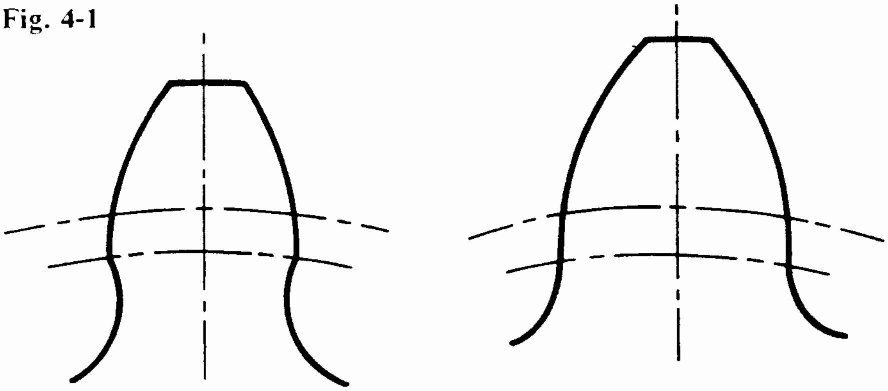
$$D_R = \frac{1}{P}\,(N - 6.9758) + (2.7475 \times t) \qquad \text{Equ. 4-5}$$

$$= \frac{1}{48}\,(9 - 6.9758) + (2.7475 \times .0406)$$

$$= .1537$$

If a standard gear has a small number of teeth, the base circle will be greater than the root diameter. Since no tooth action can take place below the base circle, that part of the tooth inside the base circle is non-functional. To accommodate the tip of the mating tooth as it sweeps around, this non-functional lower portion is undercut. Undercutting is bad in that it weakens the tooth, causes wear and inhibits continuity of action.

Fig. 4-1



(a) (b)

In Fig. 4-1 are shown the teeth from two gears having the same number of teeth, the same diametral pitch and the same PGT-1 tooth form. Tooth (a) has the standard thickness of $\frac{\pi}{2P}$. Tooth (b) has a thickness increased above standard. Tooth (b) is clearly the better of the two: it has more functional profile, and the undercut is reduced significantly.

The teeth of all gears having small numbers of teeth should have tooth thicknesses that are greater than standard. The minimum thicknesses required to insure adequate involute profiles and the avoidance of objectionable undercutting in the teeth of gears having the PGT tooth forms are obtained by use of the equations in Table 4-3.

Table 4-3

Tooth form	Minimum circular tooth thickness	**Equ. No.**
PGT-1	$t = \frac{2.3329 - (.0426 \times N)}{P}$	**4-10**
PGT-2	$t = \frac{2.4793 - (.0426 \times N)}{P}$	**4-11**
PGT-3	$t = \frac{2.5768 - (.0426 \times N)}{P}$	**4-12**
PGT-4	$t = \frac{2.6751 - (.0426 \times N)}{P}$	**4-13**

where:
t = minimum circular tooth thickness
N = number of teeth
P = diametral pitch

Example 4-4

A gear has 10 teeth, a diametral pitch of 36 and the PGT-2 tooth form. Find the minimum circular tooth thickness and the outside and root diameters.

(1) Minimum tooth thickness

$$t = \frac{2.4793 - (.0426 \times N)}{P} \qquad \text{Equ. 4-11}$$

$$N = 10 \quad P = 36$$

$$t = \frac{2.4793 - (.0426 \times 10)}{36}$$

$$= .0570$$

(2) Outside diameter

$$D_o = \frac{1}{P}(N - 2.0158) + (2.7475 \times t) \qquad \text{Equ. 4-2}$$

$$= \frac{1}{36}(10 - 2.0158) + (2.7475 \times .0570)$$

$$= .3784$$

Check for maximum possible outside diameter

$$D_o(\text{max}) = \frac{N \times .93969262}{P \times 1.017 \times \cos\phi_1} \qquad \text{Equ. 4-9}$$

$$\text{inv}\,\phi_1 = \frac{t \times P}{N} + .01490438$$

$$= \frac{.0570 \times 36}{10} + .01490438$$

$$= .22010438$$

$$\phi_1 = 45.311855°$$

$$\cos\phi_1 = .70324762$$

$$D_o(\text{max}) = \frac{10 \times .93969262}{36 \times 1.017 \times .70324762}$$

$$= .3650$$

The outside diameter to be specified for this gear is .3650, as it is the lesser of the two.

(3) Root diameter

$$D_R = \frac{1}{P}(N - 7.2758) + (2.7475 \times t)$$

$$= \frac{1}{36}(10 - 7.2758) + (2.7475 \times .057)$$

$$= .2323$$

For ready reference when preliminary design work is being done, the minimum tooth thicknesses and the corresponding outside and root diameters for the four PGT tooth forms are listed in Tables 4-4, 4-5, 4-6 and 4-7. The values given are for gears having 6 teeth up to a number such that no undesirable amount of undercutting is present at the standard tooth thickness. If a gear has a tooth thickness less than standard and a number of teeth more than the maximum given in the appropriate table, the equation from Table 4-3 must be used to insure that the tooth thickness is not less than the allowable minimum.

The values in the tables are for 1.0 diametral pitch. For other gears divide by the diametral pitch specified.

Table 4-4 (PGT-1 TOOTH FORM)

Number of teeth	Minimum Circular tooth thickness	Outside diameter	Root diameter
6	2.0773	8.9254	4.7316
7	2.0347	9.9477	5.6145
8	1.9921	10.9578	6.4975
9	1.9495	11.9577	7.3805
10	1.9069	12.9234	8.2634
11	1.8643	13.8064	9.1464
12	1.8217	14.6893	10.0293
13	1.7791	15.5723	10.9123
14	1.7365	16.4553	11.7952
15	1.6939	17.3382	12.6782
16	1.6513	18.2212	13.5611
17	1.6087	19.1041	14.4441
18	1.5708	20.0000	15.3400

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Table 4-5 (PGT-2 TOOTH FORM)

Number of teeth	Minimum Circular tooth thickness	Outside diameter	Root diameter
6	2.2237	9.0970	4.8338
7	2.1811	10.1247	5.7168
8	2.1385	11.1398	6.5997
9	2.0959	12.1446	7.4827
10	2.0533	13.1405	8.3656
11	2.0107	14.1287	9.2486
12	1.9681	15.1100	10.1316
13	1.9255	16.0852	11.0145
14	1.8829	17.0549	11.8975
15	1.8403	18.0196	12.7804
16	1.7977	18.9234	13.6634
17	1.7551	19.8064	14.5463
18	1.7125	20.6893	15.4293
19	1.6699	21.5723	16.3123
20	1.6273	22.4552	17.1952
21	1.5847	23.3382	18.0782
22	1.5708	24.3000	19.0400

Table 4-6 (PGT-3 TOOTH FORM)

Number of teeth	Minimum Circular tooth thickness	Outside diameter	Root diameter
6	2.3212	9.2103	4.9017
7	2.2786	10.2413	5.7846
8	2.2360	11.2597	6.6676
9	2.1934	12.2677	7.5505
10	2.1508	13.2666	8.4335
11	2.1082	14.2577	9.3165
12	2.0656	15.2419	10.1994
13	2.0230	16.2200	11.0824
14	1.9804	17.1924	11.9653
15	1.9378	18.1598	12.8483
16	1.8952	19.1226	13.7313
17	1.8526	20.0810	14.6142
18	1.8100	21.0355	15.4972
19	1.7674	21.9863	16.3801
20	1.7248	22.9231	17.2631
21	1.6822	23.8061	18.1460
22	1.6396	24.6890	19.0290
23	1.5970	25.5720	19.9120
24	1.5708	26.5000	20.8400

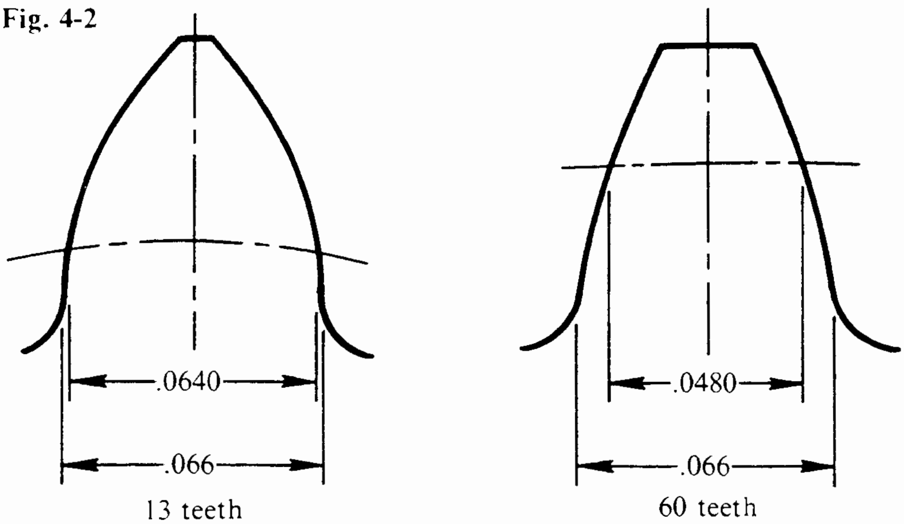
Table 4-7 (PGT-4 TOOTH FORM)

Number of teeth	Minimum Circular tooth thickness	Outside diameter	Root diameter
6	2.4195	9.3236	4.9718
7	2.3769	10.3580	5.8547
8	2.3343	11.3795	6.7377
9	2.2917	12.3907	7.6206
10	2.2491	13.3902	8.5036
11	2.2065	14.3867	9.3866
12	2.1639	15.3737	10.2695
13	2.1213	16.3545	11.1525
14	2.0787	17.3296	12.0354
15	2.0361	18.2996	12.9184
16	1.9935	19.2650	13.8013
17	1.9509	20.2260	14.6843
18	1.9083	21.1830	15.5673
19	1.8657	22.1363	16.4502
20	1.8231	23.0861	17.3332
21	1.7805	24.0326	18.2161
22	1.7379	24.9760	19.0991
23	1.6953	25.9164	19.9820
24	1.6527	26.8541	20.8650
25	1.6101	27.7891	21.7479
26	1.5708	28.7000	22.6400

To get the most out of the comparatively low tensile strengths of the plastics it is essential that gearing in power drives be so designed that the teeth of the gears be as strong as possible in terms of their geometry. It is just as essential that the teeth of a pair of mating gears be equal in strength.

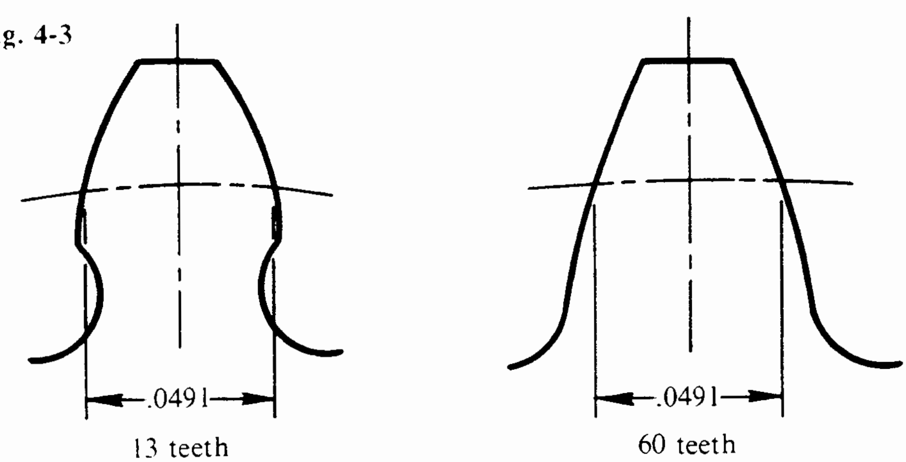
A gear tooth is, in effect, a short cantilevered beam. For the teeth of two mating gears to have equal strength, it follows that they should have equal thicknesses at their roots—where the root radii are tangent to the flanks of the teeth. To so design gear teeth for balanced strength is, admittedly, an over-simplification, because the stresses to which teeth are subjected are complex; but it is a design procedure of proven merit. It has been employed to design plastics gearing in diverse power applications, all operating successfully in the field.

Fig. 4-2



In Fig. 4-2 are shown enlargements of the profiles of the tooth of a pinion having 13 teeth and the tooth of a gear having 60 teeth. The diametral pitch is 32, and the teeth have the PGT-1 tooth form. The teeth are designed to have balanced strength: both have the same thickness of .066 where the root radii are tangent to the flanks. The standard circular tooth thickness corresponding to a diametral pitch of 32 is .0491. To achieve equal strength, the circular tooth thickness of the pinion is increased to .0640 and that of the gear is reduced to .0480.

Fig. 4-3



Shown in Fig. 4-3, by way of comparison, are the same two teeth, but each having the standard tooth thickness of .0491. The pinion tooth is much weaker than that of the gear, so much so that the pinion would be capable of transmitting only sixty percent of the load that could be handled by the gear.

If the teeth of the pinion were designed to have the minimum tooth thickness given by Equation 4-10, they would not have the undesirable amount of undercut shown in Fig. 4-3, and would be stronger, although not equal in strength to the teeth of the gear. Fig. 4-3 illustrates the importance of arriving at the right tooth thicknesses for the gearing under consideration, and again emphasizes the necessity always to regard standard tooth thicknesses as nothing more than references.

Equations 4-14, 4-15 and 4-16 are the equations used to calculate the circular tooth thicknesses of a pair of mating gears in order that the gears may have balanced tooth strength. The equations are valid only for gears having the PGT-1 tooth form. The longer teeth of the PGT-2, PGT-3 and PGT-4 tooth forms are not normally required for the coarser pitch gearing employed in power drives.

The equations provide answers for any ratio and all combinations of numbers of teeth, but it is advisable that the pinion in a power drive have at least 12 teeth. Pinions having less than 12 teeth will have reduced outside diameters (Equ. 4-9).

Equations 4-14 and 4-15 give specific values for the tooth thicknesses required of both the pinion and gear. In using Equation 4-16 it is necessary first to choose a tooth thickness for the gear. The equation then gives the tooth thickness of the pinion. In referring to pinions and gears it is understood that the pinion is the gear of a mating pair having the lesser number of teeth.

Equation 4-14

Pinion and gear both having less than 35 teeth

$$t_1 = \frac{2.3329 - (.0219 \times N_1)}{P} \quad \text{(a)}$$

$$t_2 = \frac{2.3329 - (.0219 \times N_2)}{P} \quad \text{(b)}$$

Equation 4-15

Pinion having less than 35 teeth and gear having 35 teeth or more

$$t_1 = \frac{2.3329 - (.0219 \times N_1)}{P} \quad \text{(a)}$$

$$t_2 = \frac{N_2}{P}\left(\frac{2.1922 - (.0066 \times N_1)}{N_2 - 2.0938} + \operatorname{inv}\phi_2 - .01490438\right) \quad \text{(b)}$$

Equation 4-16

Pinion and gear both having 35 teeth or more

$$t_1 = \left(\frac{N_1 \times (N_2 - 2.0938)}{N_1 - 2.0938}\right)\left(\frac{t_2}{N_2} + \frac{.01490438 - \operatorname{inv}\phi_2}{P}\right) - N_1\left(\frac{.01490438 - \operatorname{inv}\phi_1}{P}\right)$$

where

t_1 = circular tooth thickness of pinion
t_2 = circular tooth thickness of gear
N_1 = number of teeth in pinion
N_2 = number of teeth in gear
P = diametral pitch

$$\phi_1 = \cos^{-1}\left(\frac{.93969262 \times N_1}{N_1 - 2.0938}\right)$$

$$\phi_2 = \cos^{-1}\left(\frac{.93969262 \times N_2}{N_2 - 2.0938}\right)$$

Example 4-5

A pinion having 15 teeth drives a gear having 30 teeth. The diametral pitch is 18 and the PGT-1 tooth form is specified. Determine the circular tooth thicknesses of both the pinion and gear for balanced tooth strength.

$$t_1 = \frac{2.3329 - (.0219 \times N_1)}{P} \qquad \text{Equ. 4-14(a)}$$

$N_1 = 15 \quad P = 18$

$$t_1 = \frac{2.3329 - (.0219 \times 15)}{18} = .1114$$

$$t_2 = \frac{2.3329 - (.0219 \times N_2)}{P} \qquad \text{Equ. 4-14(b)}$$

$N_2 = 30 \quad P = 18$

$$t_2 = \frac{2.3329 - (.0219 \times 30)}{18} = .0931$$

Circular tooth thickness of the pinion = .1114
Circular tooth thickness of the gear = .0931

Example 4-6

A pinion having 12 teeth drives a gear having 48 teeth. The diametral pitch is 16 and the PGT-1 tooth form is specified. Determine the circular tooth thicknesses of the pinion and gear for balanced tooth strength.

$$t_1 = \frac{2.3329 - (.0219 \times N_1)}{P} \qquad \text{Equ. 4-15(a)}$$

$N_1 = 12 \quad P = 16$

$$t_1 = \frac{2.3329 - (.0219 \times 12)}{16} = .1294$$

$$t_2 = \frac{N_2}{P}\left(\frac{2.1922 - (.0066 \times N_1)}{N_2 - 2.0938} + \text{inv}\,\phi_2 - .01490438\right) \qquad \text{Equ. 4-15(b)}$$

$N_1 = 12 \quad N_2 = 48 \quad P = 16$

$$\phi_2 = \cos^{-1}\left(\frac{.93969262 \times N_2}{N_2 - 2.0938}\right)$$

$$= \cos^{-1}\left(\frac{.93964262 \times 48}{48 - 2.0938}\right)$$

$$= 10.718632^\circ$$

$\text{inv}\,\phi_2 = .00221336$

$$t_2 = \frac{48}{16}\left(\frac{2.1922 - (.0066 \times 12)}{48 - 2.0938} + .00221336 - .01490438\right) = .1000$$

Circular tooth thickness of the pinion = .1294
Circular tooth thickness of the gear = .1000

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Example 4-7

A pinion having 37 teeth drives a gear having 74 teeth. The diametral pitch is 20 and the PGT-1 tooth form is specified. Determine the circular tooth thicknesses of the pinion and gear for balanced tooth strength.

$$t_1 = \left(\frac{N_1 \times (N_2 - 2.0938)}{N_1 - 2.0938}\right)\left(\frac{t_2}{N_2} + \frac{.01490438 - \operatorname{inv}\phi_2}{P}\right) - N_1\left(\frac{.01490438 - \operatorname{inv}\phi_1}{P}\right) \qquad \text{Equ. 4-16}$$

$N_1 = 37 \quad N_2 = 74 \quad P = 20$

$$\phi_1 = \cos^{-1}\left(\frac{.93969262 \times N_1}{N_1 - 2.0938}\right)$$

$$= \cos^{-1}\left(\frac{.93969262 \times 37}{37 - 2.0938}\right)$$

$$= 5.088569^\circ$$

$$\operatorname{inv}\phi_1 = .00023424$$

$$\phi_2 = \cos^{-1}\left(\frac{.93969262 \times N_2}{N_2 - 2.0938}\right)$$

$$= \cos^{-1}\left(\frac{.93969262 \times 74}{74 - 2.0938}\right)$$

$$= 14.747949^\circ$$

$$\operatorname{inv}\phi_2 = .00583948$$

Let t_2 = standard circular tooth thickness $= \frac{\pi}{2P} = \frac{3.1415926}{2 \times 20} = .0785$

$$t_1 = \left(\frac{37 \times (74 - 2.0938)}{37 - 2.0938}\right)\left(\frac{.0785}{74} + \frac{.01490438 - .00583948}{20}\right) - 37 \times \left(\frac{.01490438 - .00023424}{20}\right)$$

$$= .0883$$

Circular tooth thickness of the pinion = .0883
Circular tooth thickness of the gear = .0785

CENTER DISTANCE

When two standard gears are brought into close mesh, the distance between their centers is half the sum of their standard pitch diameters and is referred to as the standard center distance. But it is rare for a pair of standard gears to be the best combination for a given drive. As was discussed in Chapter 1, a pair of involute gears will function at widely varying center distances. The center distance at which a pair of mating gears are to operate should be regarded as a variable that can be manipulated to achieve the best possible drive.

The center distance at which a pair of PGT tooth form gears are in close mesh is determined by use of Equation 5-1.

Equation 5-1

$$C = \frac{(N_1 + N_2) \times .46984631}{P \times \cos\phi_1}$$

where:

C = close mesh center distance
N_1 = number of teeth in first gear
N_2 = number of teeth in second gear
P = diametral pitch
ϕ_1 = angle whose involute is $\frac{P(t_1 + t_2) - \pi}{N_1 + N_2} + .01490438$

where:

t_1 = circular tooth thickness of 1st gear
t_2 = circular tooth thickness of 2nd gear
π = 3.1415926

Example 5-1

Two mating gears have the basic specifications given below. Find the center distance at which they will be in close mesh.

	1st Gear	2nd Gear
Number of teeth	15	60
Diametral pitch	32	32
Tooth form	PGT-1	PGT-1
Standard pitch diameter	.4688	1.8750
Calc. cir. tooth thickness on std. pitch circle	.0539	.0491

$$C = \frac{(N_1 + N_2) \times .46984631}{P \times \cos\phi_1} \qquad \text{Equ. 5-1}$$

$N_1 = 15 \quad N_2 = 60 \quad P = 32$

$$\text{inv}\,\phi_1 = \frac{P(t_1 + t_2) - \pi}{N_1 + N_2} + .01490438$$

$t_1 = .0539 \quad t_2 = .0491 \quad \pi = 3.1415926$

$$\text{inv}\,\phi_1 = \frac{32(.0539 + .0491) - 3.1415926}{15 + 60} + .01490438$$

$$= .01696315$$

$$\phi_1 = 20.850399°$$

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$$\phi_1 = 20.850399^\circ$$

$$\cos\phi_1 = .93451295$$

$$C = \frac{(15 + 60) \times .46984631}{32 \times .93451295}$$

$$= 1.17837$$

The close mesh center distance as determined by Equ. 5-1 assumes perfection in the gears and any bearings employed, and assumes the gears and the housing in which they are mounted to be made of stable materials. Runout in the gears and bearings, and changes in the sizes of the gears and housing with changes of the environment, will cause the close mesh center distance to vary from a high to a low. The minimum operating center distance specified for the housing must be equal to, or greater than, the maximum close mesh center distance; otherwise the gears may bind.

Knowing the accuracies specified for the gears and bearings, the materials of which the gears and housing are to be made, and the nature of the environment, the amount by which the minimum operating center distance must exceed the close mesh center distance can be calculated.

The errors that can be present in a gear are fully discussed in Chapter 4 of "Accurate Molded Plastic Gears". As explained, the sum of the errors in the teeth is called the tooth-to-tooth composite error, and that error, plus the runout in the gear, is called the total composite error. It was further explained that a system has been devised by the American Gear Manufacturers Association whereby a gear is classified by number in accordance with the accuracy required of it. The system is presented, in detail, in the "Gear Handbook,"American Gear Manufacturers Association Publication 2000-A88.

Suppose it were specified that the gears in Example 5-1 were to be to the accuracies required of AGMA Quality Number Q7. From the "Gear Handbook" it is found that the first gear is permitted a maximum total composite tolerance of .0031 and the second, a maximum total composite tolerance of .0034. The close mesh center distance was calculated to be 1.17837. If the errors in the gears were at the maximum allowed by the tolerances, these errors would cause the close mesh center distance to go from a high of 1.18162 to a low of 1.17512. To allow for these maximum permissable errors, the operating center distance must be not less than 1.18162.

To allow for the errors in each of two mating gears, the minimum operating center distance must exceed the calculated close mesh center distance by half the sum of the total composite tolerances of the gears.

Because the coefficients of linear thermal expansion of the plastics are relatively high, and because some plastics expand as they pick up moisture, these factors must also be taken into consideration in arriving at the minimum operating center distance.

Finally, if the gears are mounted on shafts running in bearings, the maximum allowable runout of the bearings must be taken into account.

Assuming that the gears are inspected in the dry, as-molded, condition, and at a room temperature of 70° F, the total amount the minimum operating center distance must exceed the calculated close mesh center distance is expressed as follows:

Equation 5-2

$$\Delta_C = \frac{TCT_1 + TCT_2}{2} + C\left[(T-70)\left(\frac{COEF_1 \times N_1}{N_1+N_2} + \frac{COEF_2 \times N_2}{N_1+N_2} - COEF_H\right) + \left(\frac{M_1 \times N_1}{N_1+N_2} + \frac{M_2 \times N_2}{N_1+N_2} - M_H\right)\right] + \frac{TIR_1 + TIR_2}{2}$$

where:

Δ_C = required increase in center distance
TCT_1 = maximum total composite tolerance of 1st gear
TCT_2 = maximum total composite tolerance of 2nd gear
C = close mesh center distance
T = maximum temperature to which gears will be subjected (° F)
$COEF_1$ = coefficient of linear thermal expansion of material of 1st gear (in/in/° F)
$COEF_2$ = coefficient of linear thermal expansion of material of 2nd gear (in/in/° F)
$COEF_H$ = coefficient of linear thermal expansion of material of housing (in/in/° F)
N_1 = number of teeth in 1st gear
N_2 = number of teeth in 2nd gear
M_1 = expansion due to moisture pick-up of material of 1st gear (in/in)
M_2 = expansion due to moisture pick-up of material of 2nd gear (in/in)
M_H = expansion due to moisture pick-up of material of the housing (in/in)
TIR_1 = maximum allowable runout of bearing of 1st gear
TIR_2 = maximum allowable runout of bearing of 2nd gear

The coefficients of linear thermal expansion of the plastics are to be found in the properties charts issued by the plastics suppliers. Rates of water absorption are also included in these charts, but information about expansion due to water absorption is not so readily available. In the case of applications where the gears will not be exposed immediately to conditions of high humidity, the expansion of most plastics is small and gradual and can be discounted, because it is off-set by the equally small and gradual shrinkage that occurs as molding stresses are relieved. If the gears are to be molded of one of the more hygroscopic plastics, and if conditions are such that there could be a high percentage of water absorption, it is advisable to consult with the technical department of the plastic supplier to determine what allowance, in terms of in/in, should be allowed.

Example 5-2

The first gear in Example 5-1 has 15 teeth and the second 60. The gears are to be to the accuracy required of AGMA Quality Number Q7. The maximum total composite tolerances are .0031 and .0034 respectively. The first gear is to be molded of a nylon having a coefficient of linear thermal expansion of 5.0×10^{-5} in/in/° F and the second of an acetal having a coefficient of linear thermal expansion of 4.5×10^{-5} in/in/° F. The environment in which the gears will operate is such that it has been determined that expansion of the nylon due to water absorption could be .003 in/in and that of the acetal .0005 in/in. The housing is made of an aluminum having a coefficient of linear thermal expansion of 1.0×10^{-5} in/in/° F. The gear shafts run in bearings concentric to .0005 *T.I.R*. The maximum temperature to which the gears will be subjected is 150° F. Find the minimum operating center distance to be specified on the drawing of the housing.

It has already been determined, in Example 5-1, that the gears will be in close mesh at a center distance of 1.17837. The calculation employed to arrive at this center distance allowed for no errors in the gears, assumed them to be in the dry, as-molded condition, and at a temperature of 70° F.

Find Δ_C

$$\Delta_C = \frac{TCT_1 + TCT_2}{2} + C\left[(T-70)\left(\frac{COEF_1 \times N_1}{N_1+N_2} + \frac{COEF_2 \times N_2}{N_1+N_2} - COEF_H\right) + \left(\frac{M_1 \times N_1}{N_1+N_2} + \frac{M_2 \times N_2}{N_1+N_2} - M_H\right)\right] + \frac{TIR_1 + TIR_2}{2}$$

Equ. 5-2

$TCT_1 = .0031$
$TCT_2 = .0034$
$C = 1.17837$
$T = 150$
$COEF_1 = 5.0 \times 10^{-5} = .00005$
$COEF_2 = 4.5 \times 10^{-5} = .000045$
$COEF_H = 1.0 \times 10^{-5} = .00001$
$N_1 = 15$ $M_H = .0000$
$N_2 = 60$ $TIR_1 = .0005$
$M_1 = .003$ $TIR_2 = .0005$
$M_2 = .0005$

$$\Delta_C = \frac{.0031 + .0034}{2} + 1.17837\left[(150-70)\left(\frac{.00005 \times 15}{15+60} + \frac{.000045 \times 60}{15+60} - .00001\right) + \left(\frac{.003 \times 15}{15+60} + \frac{.0005 \times 60}{15+60} - .0000\right)\right] + \frac{.0005 + .0005}{2}$$

$= .00325 + .00457 + .0005$

$= .00832$

Minimum operating center distance $= 1.17837 + .00832$
$= 1.1867$

In example 5-1 the tooth thicknesses, which were specified, determined the close mesh center distance. When the minimum operating center distance is fixed by requirements of the mechanism of which the gears are part, it then becomes necessary to establish tooth thicknesses such that the close mesh center distance is less than the minimum operating center distance by Δ_C.

Δ_C is again calculated by use of Equation 5-2, but the value given to C is now that of the minimum operating center distance. This will introduce an error, because C should have the value of the unknown close mesh center distance, but an error so small it can be considered negligible.

Δ_C is then subtracted from the minimum operating center distance. The answer is the close mesh center distance used in Equation 5-3 to get the sum of the tooth thicknesses for a given close mesh center distance.

Equation 5-3

$$t_1 + t_2 = \frac{(N_1 + N_2)(\text{inv}\,\phi_1 - .01490438) + \pi}{P}$$

where:
t_1 = circular tooth thickness of 1st gear
t_2 = circular tooth thickness of 2nd gear
N_1 = number of teeth in 1st gear
N_2 = number of teeth in 2nd gear
$\pi = 3.1415926$
P = diametral pitch

$$\phi_1 = \cos^{-1}\left[\frac{(N_1 + N_2) \times .46984631}{P \times C}\right]$$

where:
C = close mesh center distance.

Example 5-3

The gears in Example 5-1 and 5-2 are required to operate at a fixed minimum operating center distance of 1.1720. Determine the circular tooth thickness of each gear.

Substituting 1.1720 for 1.17836 as the value for C in Equation 5-2,

$$\Delta_C = \frac{.0031+.0034}{2} + 1.1720\left[(150-70)\left(\frac{.00005\times15}{15+60}+\frac{.000045\times60}{15+60}-.00001\right)+\left(\frac{.003\times15}{15+60}+\frac{.0005\times60}{15+60}-.0000\right)\right]+\frac{.0005+.0005}{2}$$

$$= .00325 + .00455 + .0005$$

$$= .0083$$

Close mesh center distance = 1.1720 − .0083
= 1.1637

The tooth thicknesses must be such that if the gears and bearings were perfect, and the gears and housing made of stable materials, the gears would be in close mesh at 1.1637. The sum of the tooth thicknesses for this close mesh center distance is calculated by employing Equation 5-3.

$$t_1 + t_2 = \frac{(N_1+N_2)(\text{inv}\,\phi_1 - .01490438)+\pi}{P} \qquad \text{Equ. 5-3}$$

$N_1 = 15 \quad N_2 = 60 \quad \pi = 3.1415926 \quad P = 32$

$$\phi_1 = \cos^{-1}\left[\frac{(N_1+N_2)\times .46984631}{P \times C}\right]$$

$C = 1.1637$

$$\phi_1 = \cos^{-1}\left[\frac{(15+60)\times .46984631}{32\times 1.1637}\right]$$

$$= \cos^{-1}(.94629397)$$

$$= 18.863065^\circ$$

$\text{inv}\,\phi_1 = .01243389$

$$t_1 + t_2 = \frac{(15+60)(.01243389 - .01490438) + 3.1415926}{32}$$

$$= .0924$$

From Table 4-4, a gear having 15 teeth, a diametral pitch of 32, and the PGT-1 tooth form is required to have a minimum circular tooth thickness of .0529. Allowing for a tolerance of, say, + .0010 − .0000, the permissable maximum becomes .0539, and the maximum tooth thickness of the second gear becomes .0924 − .0539 = .0385.

The standard circular tooth thickness of a gear having a diametral pitch of 32 is .0491. The reduction to .0385, although large, is acceptable, as the teeth will have no undercut. But consider the problem encountered in the next example.

Example 5-4

Suppose the second gear has 18 teeth, and suppose the minimum operating center distance is fixed at .5156, which is the standard center distance for two gears having 15 and 18 teeth, respectively, and a diametral pitch of 32.

The total composite tolerance of the second gear is now .0031

$$\Delta_C = \frac{.0031+.0031}{2} + .5156\left[(150-70)\left(\frac{.00005\times15}{15+18}+\frac{.000045\times18}{15+18}-.00001\right)+\left(\frac{.003\times15}{15+18}+\frac{.0005\times18}{15+18}-.0000\right)\right]+\frac{.0005+.0005}{2}$$

$$= .0031 + .00238 + .0005$$

$$= .00598$$

Close mesh center distance, $C = .5156 - .00598$
$= .5096$

$$t_1 + t_2 = \frac{(15+18)(\text{inv}\,\phi_1 - .01490438) + 3.1415926}{32}$$

$$\phi_1 = \cos^{-1}\left[\frac{(15+18)\times .46984631}{32\times .5096}\right]$$

$$= \cos^{-1}(.95080260)$$

$$= 18.047020°$$

$$\text{inv}\,\phi_1 = .01084731$$

$$t_1 + t_2 = \frac{(15+18)(.01084731 - .01490438) + 3.1415926}{32}$$

$$= .0940$$

The tooth thickness of the second gear is .0940 − .0539 = .0401. But, from Table 4-4, a gear having 18 teeth must have a tooth thickness of not less than .0491 to avoid an undesirable amount of undercut.

The tooth thickness of the first gear could be reduced to, say, .0490 and that of the second to .0450. The gears would function, but the action would not be as satisfactory as it could be. A better solution is to specify a finer diametral pitch.

Example 5-5

Let the diametral pitch of the gears in Example 5-4 be changed from 32 to 32.8. Find the circular tooth thicknesses.

Both gears now have a total composite tolerance of .0030.

$$\Delta_C = .0030 + .00238 + .0005$$

$$= .00588$$

Close mesh center distance, $C = .5156 - .00588$
$= .5097$

$$t_1 + t_2 = \frac{(15 + 18)(\text{inv}\,\phi_1 - .01490438) + 3.1415926}{32.8}$$

$$\phi_1 = \cos^{-1}\left[\frac{(15 + 18) \times .46984631}{32.8 \times .5097}\right]$$

$$= \cos^{-1}(.92743031)$$

$$= 21.962275°$$

$$\text{inv}\,\phi_1 = .01994651$$

$$t_1 + t_2 = \frac{(15 + 18)(.01994651 - .01490438) + 3.1415926}{32.8}$$

$$= .1009$$

From Table 4-4 the minimum tooth thickness of the first gear, which has 15 teeth and a diametral pitch of 32.8, is .0516, say, .0526 + .0000 − .0010. The sum of the tooth thicknesses is .1009; therefore, the tooth thickness of the second gear is .1009 − .0526 = .0483. From Table 4-4 the minimum tooth thickness of a gear having 18 teeth and a diametral pitch of 32.8 is .0479.

The diametral pitch of 32.8 was arrived at by trial, from Equations 5-2 and 5-3. This example demonstrates the freedom accorded the designer by the molding process. A diametral pitch of 32.8 is required, so that is what is specified. There in nothing to be gained economically, and much to be lost, by a rigid adherence to what are considered to be the “standard” diametral pitches.

CONTACT RATIO

When one gear drives another, one pair of mating teeth must take over the drive before the preceding pair part company. The average number of teeth always in contact is called the contact ratio.

Fig. 6-1

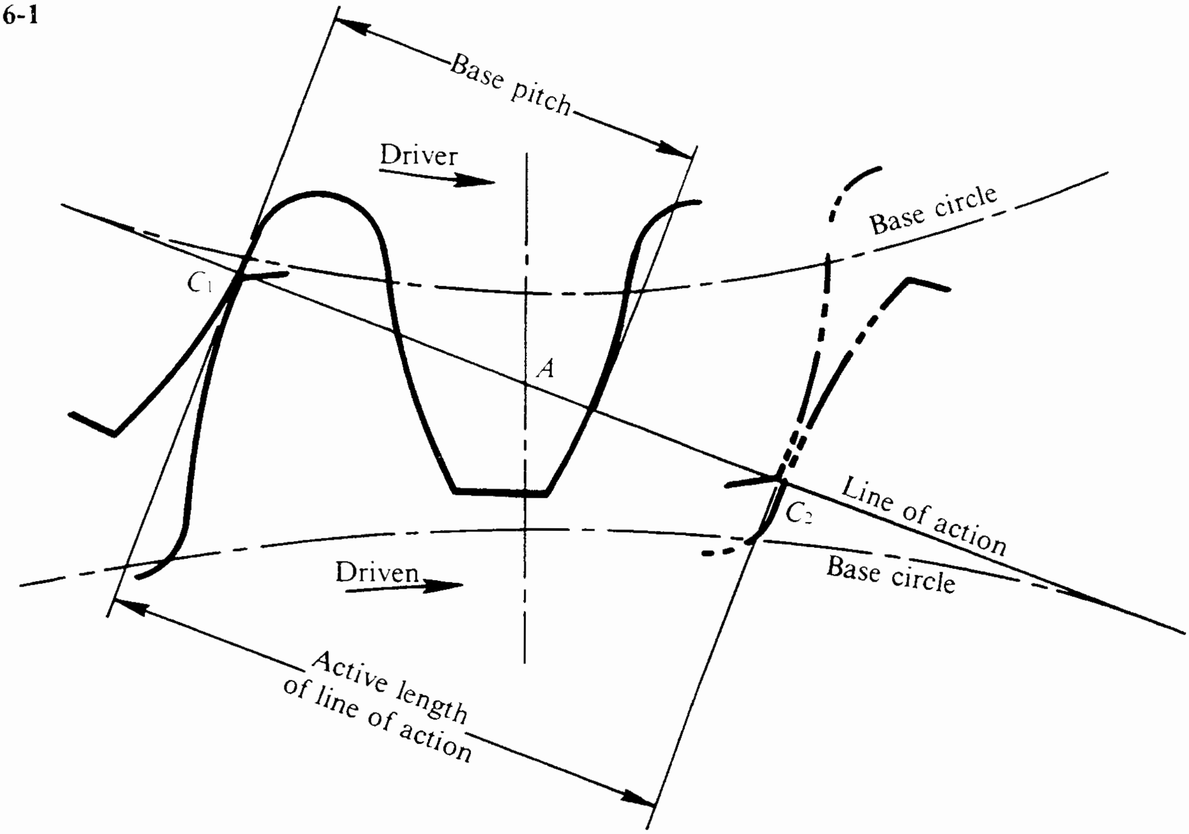


Fig. 6-1 shows the profiles of a pair of mating teeth. The tooth of the driving gear first makes contact with the tooth of the driven gear at C_1. Contact continues along the line of action, through the pitch point A, until the teeth break contact at C_2. The distance between two involutes along the line of action is equal to the base pitch. The active length of the line of action divided by the base pitch is called the contact ratio.

There can be no continuity of action if the active length of the line of action is less than the base pitch, therefore, the contact ratio must always be greater than 1.0. For an even and smooth transfer of load from one pair of mating teeth to the next it should be at least 1.2.

The contact between a pair of mating teeth is a combination of sliding and rolling. From C_1 to A the driving tooth is sliding into the driven tooth. At the pitch point, A, there is a momentary rolling and then, from A to C_2, the driving tooth slides out of engagement with the driven tooth. In gearing terminology, there is an approach action from C_1 to A and a recess action from A to C_2.

During approach action, the driven tooth tends to dig into the driving tooth, causing substantially greater wear than occurs during recess action, when a smooth withdrawal is taking place. Tests have demonstrated that the coefficient of friction occuring during the approach action can be twice that which occurs during the recess action. A pair of gears should be designed to have as much recess action as other considerations will allow.

SPUR & HELICAL GEARS

The recess and approach increments of the active length of the line of action are determined by use of Equations 6-1 and 6-2. The equations can be used for all gears having any one of the four PGT tooth forms, provided always, that the teeth of the gears have the minimum thicknesses required to avoid an undesirable amount of undercut. If the circular tooth thickness on the standard pitch circle of one or both of a pair of mating gears is less than given by Equations 4-10 through 4-13, the answers given by Equations 6-1 and 6-2 will be erroneously large.

The contact ratio is the sum of the recess and approach increments divided by the base pitch.

$$D_b = D \times \cos\phi$$

$$\text{Base pitch} = \frac{D \times \cos\phi \times \pi}{N}$$

$$D = \frac{N}{P}$$

$$\text{Base pitch} = \frac{\cos\phi \times \pi}{P}$$

$$\phi = \text{pressure angle} = 20^\circ$$

$$\text{Base pitch} = \frac{.93969262 \times 3.1415926}{P}$$

$$= 2.9521314 \div P$$

The contact ratio of a pair of mating gears is the sum of the answers given by Equations 6-1 and 6-2 divided by $(2.9521314 \div P)$.

Equation 6-1

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2}$$

Equation 6-2

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2}$$

Where:

RA = recess increment of line of action
AA = approach increment of line of action
D_{o1} = outside diameter of driving gear
D_{o2} = outside diameter of driven gear
N_1 = number of teeth in driving gear
N_2 = number of teeth in driven gear
C = operating center distance
$Y = \dfrac{P}{.93969262}$
P = diametral pitch

Equation 6-3

$$CR = (RA + AA) \div (2.9521314 \div P)$$

Where
CR = contact ratio
RA = recess increment as given by Equ. 6-1
AA = approach increment as given by Equ. 6-2
P = diametral pitch.

Example 6-1

A pair of mating gears have the data listed below. Determine the recess action, the approach action and the contact ratio. The first gear drives the second.

	First gear (driver)	Second gear (driven)
Number of teeth	20	60
Diametral pitch	24	24
Tooth form	PGT-1	PGT-1
Circular tooth thickness	.0746	.0578
Outside diameter	.942	2.562
Operating center distance	1.6750	

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 6-1}$$

$D_{o1} = .942 \quad N_1 = 20 \quad N_2 = 60 \quad C = 1.675$

$$Y = \frac{P}{.93969262}$$

$$P = 24$$

$$Y = \frac{24}{.93969262} = 25.54027$$

$$RA = \frac{.942}{2}\sqrt{1 - \left(\frac{20}{.942 \times 25.54027}\right)^2} - \frac{1.675 \times 20}{20 + 60}\sqrt{1 - \left(\frac{20 + 60}{2 \times 1.675 \times 25.54027}\right)^2}$$

$$= .26179873 - .14848951 = .11330922$$

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 6-2}$$

$D_{o2} = 2.562 \quad N_1 = 20 \quad N_2 = 60 \quad C = 1.675 \quad Y = 25.54027$

$$AA = \frac{2.562}{2}\sqrt{1 - \left(\frac{60}{2.562 \times 25.54027}\right)^2} - \frac{1.675 \times 60}{20 + 60}\sqrt{1 - \left(\frac{20 + 60}{2 \times 1.675 \times 25.54027}\right)^2}$$

$$= .51111558 - .44546854 = .06564704$$

$$CR = (RA + AA) \div (2.9521314 \div P) \qquad \text{Equ. 6-3}$$

$RA = .11330922 \quad AA = .06564704 \quad P = 24$

$CR = (.11330922 + .06564704) \div (2.9521314 \div 24) = 1.455$

$$\text{Percentage of recess action} = \frac{.11330922 \times 100}{.11330922 + .06564704} = 63.3$$

The gears in Example 6-1 have an ample contact ratio and 63 percent of the tooth action is in the recess zone of the line of contact. The drive will be smooth and even.

A study of Equations 6-1 and 6-2 will reveal that the greater the addendum of the driving gear is over that of the driven gear, the greater is the percentage of recess action. The tooth thickness of a pinion is increased above standard, either to avoid an undesirable amount of undercut or to achieve balanced tooth strength. As a consequence, the addendum of the pinion is invariably greater than that of the mating gear. Provided the drive is speed reducing, so that the pinion is driving the gear, the recess action of a pair of correctly designed involute gears will exceed fifty percent.

If the drive is speed-increasing, the gear becomes the driver, and the opposite holds true. If the gear in Example 6-1 were the driver, the approach action would then be 63 percent, resulting in a rough drive, excessive wear and low efficiency.

Fortunately, the vast majority of gear drives are speed-reducing. When a speed-increasing drive is encountered, the gears must be designed to have as much recess action as other considerations will permit.

Example 6-2

The drive in Example 6-1 is reversed so that the gear is now the driver. Redesign the pair to reduce the approach action to a more desirable percentage.

The pinion and gear in Example 6-1 are designed to have equal tooth strength. The pinion has a circular tooth thickness of .0746. This tooth thickness could be reduced to the standard thickness of .0654 without the introduction of any undesirable undercut (Table 4-4). The outside diameter would then be .917 (Equ. 4-1).

The operating center distance is unchanged at 1.6750. Therefore the sum of the circular tooth thicknesses remains at .0746 + .0578 = .1324. The circular tooth thickness of the gear is now .1324 − .0654 = .0670 and the outside diameter is 2.588 (Equ. 4-1).

Find the percentages of recess and approach action, and the contact ratio.

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 6-1}$$

$D_{o1} = 2.588 \quad N_1 = 60 \quad N_2 = 20 \quad C = 1.675 \quad Y = 25.54027$

$$RA = \frac{2.588}{2}\sqrt{1 - \left(\frac{60}{2.588 \times 25.54027}\right)^2} - \frac{1.675 \times 60}{60 + 20}\sqrt{1 - \left(\frac{60 + 20}{2 \times 1.675 \times 25.54027}\right)^2}$$

$$= .09740727$$

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 6-2}$$

$D_{o2} = .917$

$$AA = \frac{.917}{2}\sqrt{1 - \left(\frac{20}{.917 \times 25.54027}\right)^2} - \frac{1.675 \times 20}{60 + 20}\sqrt{1 - \left(\frac{60 + 20}{2 \times 1.675 \times 25.54027}\right)^2}$$

$= .09008925$

$CR = (RA + AA) \div (2.9521314 \div P)$

$RA = .09740727 \quad AA = .09008925 \quad P = 24$

$CR = (.09740727 + .09008925) \div (2.9521314 \div 24) = 1.524$

$$RA = \frac{.09740727 \times 100}{.09740727 + .09008925} = 52\%$$

The recess action is now 52 percent, and the contact ratio is more than adequate. But the pinion and gear no longer have balanced tooth strength. As a result, the load that can be transmitted is now approximately 76 percent of what could be transmitted by the pair in Example 6-1. Designing a speed-increasing drive invariably involves the acceptance of a compromise.

The input torque to an instrument gear train is frequently so low that the overall efficiency of the gearing becomes a matter of paramount importance. Their light weights and low coefficients of friction make the plastics unequalled as materials for the gears in instrument movements, provided the gears are correctly designed. A pair of mating gears must have a high percentage of recess action to achieve high efficiency and must have an adequate contact ratio to insure smooth, uniform motion.

The longer tooth forms of the PGT System should be specified for all fine-pitch instrument gears molded of the plastics. The benefits accruing from the use of these longer teeth are best demonstrated by an example.

Example 6-3

The first reduction stage of an instrument drive consists of a pinion having 16 teeth and a gear with 80 teeth. The diametral pitch is 64. The gears are housed in an aluminum die casting. The operating center distance has been fixed at .7500 ± .0020. The spindles of the gears run in bearings having a concentricity of .0005 T.I.R. Complete the design of the pinion and gear to have a high percentage of recess action and an adequate contact ratio.

Having reviewed the range of suitable materials, a plastic is chosen which offers a balance of good tensile strength, stiffness, low abrasion, and a low coefficient of friction, together with good molding characteristics and low cost. The plastic has a coefficient of thermal expansion of 5×10^{-5} in/in/°F. After a review of the environments in which the instrument may be required to operate, and after consultation with the supplier, it is decided to allow for a net expansion of .0005 in/in due to possible moisture absorption less the shrinkage following the relaxation of molding stresses. The instrument can be subjected to a temperature of 150° F. The coefficient of linear thermal expansion of the aluminum of which the housing is cast is 1.0×10^{-5} in/in/°F.

Gears can be molded of the plastic selected, in multi-cavity molds, and held without difficulty to the accuracies required of AGMA Quality Number Q7. Upon referring to the AGMA "Gear Handbook" it is found that the pinion and gear have total composite tolerances of .0023 and .0026 respectively.

(1) Find Δ_C

$$\Delta_C = \frac{TCT_1 + TCT_2}{2} + C\left[(T-70)\left(\frac{COEF_1 \times N_1}{N_1+N_2} + \frac{COEF_2 \times N_2}{N_1+N_2} - COEF_H\right) + \left(\frac{M_1 \times N_1}{N_1+N_2} + \frac{M_2 \times N_2}{N_1+N_2} - M_H\right)\right] + \frac{TIR_1 + TIR_2}{2}$$

Equ. 5-2

$COEF_1 = 5.0 \times 10^{-5}$ $COEF_2 = 5.0 \times 10^{-5}$ $COEF_H = 1.0 \times 10^{-5}$ $N_1 = 16$ $N_2 = 80$

$M_1 = .0005$ $M_2 = .0005$ $M_H = .0000$ $TIR_1 = .0005$ $TIR_2 = .0005$ $C = .750$

$$\Delta_C = \frac{.0023+.0026}{2} + .750\left[(150-70)\left(\frac{.00005\times16}{16+80} + \frac{.00005\times80}{16+80} - .00001\right) + \left(\frac{.0005\times16}{16+80} + \frac{.0005\times80}{16+80} - .0000\right)\right] + \frac{.0005+.0005}{2}$$

$= .0057$

(2) Find the sum of the circular tooth thicknesses of the pinion and gear.
Operating center distance = .7500 ± .0020
Minimum operating center distance = .7480
Close mesh center distance = .7480 − Δ_C = .7480 − .0057 = .7423

$$t_1 + t_2 = \frac{(N_1 + N_2)(\text{inv}\,\phi_1 - .01490438) + \pi}{P}$$

Equ. 5-3

$N_1 = 16$ $N_2 = 80$ $P = 64$ $\pi = 3.1415926$

$$\phi_1 = \cos^{-1}\left[\frac{(N_1 + N_2) \times .46894631}{P \times C}\right]$$

$P = 64$

$$\phi_1 = \cos^{-1}\left[\frac{(16 + 80) \times .46984631}{64 \times .7423}\right]$$

$= \cos^{-1}(.94944021)$

$= 18.297311$

$\text{inv}\,\phi_1 = .01131800$

$$t_1 + t_2 = \frac{(16 + 80)(.01131800 - .01490438) + 3.1415926}{64}$$

$= .0437$

(3) Find the circular tooth thicknesses and outside diameters of the pinion and gear.

For the purpose of this example, firs' alculate the tooth thicknesses and outside diameters on the assumption that the PGT-1 tooth form is specified.

$$t_1 = \frac{1.6513}{P}$$

Table 4-4

$P = 64$

$$t_1 = \frac{1.6513}{64}$$

$= .0258$

$$D_{o1} = \frac{18.2212}{64}$$

$$= .2847$$

$$t_1 + t_2 = .0437$$

$$t_2 = .0437 - .0258$$

$$= .0179$$

$$D_{o2} = \frac{1}{P}(N - 2.3158) + (2.7475 \times t)$$ Equ. 4-1

$$= \frac{1}{64}(80 - 2.3158) + (2.7475 \times .0179)$$

$$= 1.2630$$

(4) Find the recess action, approach action and contact ratio.

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2}$$ Equ. 6-1

$D_{o1} = .2847$ $D_{o2} = 1.2630$ $N_1 = 16$ $N_2 = 60$ $C = .7520$

(The value of C used is the maximum operating center distance allowed by the tolerance)

$$Y = \frac{P}{.93969262}$$

$$P = 64$$

$$Y = 68.1074$$

$$RA = \frac{.2847}{2}\sqrt{1 - \left(\frac{16}{.2847 \times 68.1074}\right)^2} - \frac{.7520 \times 16}{16 + 80}\sqrt{1 - \left(\frac{16 + 80}{(2 \times .7520 \times 68.1074)}\right)^2}$$

$$= .03669574$$

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2}$$ Equ. 6-2

$$= \frac{1.2630}{2}\sqrt{1 - \left(\frac{80}{1.2630 \times 68.1074}\right)^2} - \frac{.7520 \times 80}{16 + 80}\sqrt{1 - \left(\frac{16 + 80}{2 \times .7520 \times 68.1074}\right)^2}$$

$$= .01349355$$

$$CR = (RA + AA) \times .3387383 \times P$$

$RA = .03669574$ $AA = .01349355$ $P = 64$

$$CR = 1.088$$

Percentage recess action = 73

The percentage of recess action is satisfactory, but the contact ratio is too small. The contact ratio can be improved by increasing the outside diameters. This can be accomplished only by increasing the tooth thicknesses, which requires that the close mesh center distance be increased by reducing Δ_C. To reduce Δ_C, both pinion and gear must be molded of a plastic having a lower coefficient of linear thermal expansion, a lesser degree of expansion due to water absorption, and must be molded to closer tolerances than those required of AGMA Quality Number Q7.

Suppose the plastic is changed to a filled variety having a coefficient of linear thermal expansion of 3.0×10^{-5} in/in/°F and an expansion due to moisture absorption of .0003 in/in. Suppose, also, that the AGMA Quality number is specified to be No. Q9 instead of Q7. These changes reduce Δ_C from .0057 to .0032 and increase $t_1 + t_2$ from .0437 to .0454. The outside diameter of the gear can now be increased from 1.2630 to 1.2677. As a result, the contact ratio is increased from 1.088 to 1.225, an acceptable figure; although, in the process, the recess action is reduced from 73 percent to 65 percent. But the pinion and gear have become more expensive to produce. A filled plastic costs more than the unfilled variety, and to mold gears to the accuracies of AGMA Quality No. Q9 requires very close process control and virtually 100 percent inspection.

Consider now what would result if no changes were made to the pinion and gear other than to specify that the tooth form be PGT-3 instead of PGT-1.

(5) Find the circular tooth thicknesses and outside diameters of the pinion and gear.

$$t_1 = \frac{1.8952}{64} \qquad \text{Table 4-6}$$

$$= .0296$$

$$D_{o1} = \frac{19.1226}{64}$$

$$= .2988$$

$$t_1 + t_2 = .0437$$

$$t_2 = .0437 - .0296$$

$$= .0141$$

$$D_{o2} = \frac{1}{64}(80 - 1.8158) + (2.7475 \times .0141)$$

$$= 1.2604$$

(6) Find the recess action, approach action and contact ratio

$$RA = \frac{.2988}{2}\sqrt{1 - \left(\frac{16}{.2988 \times 68.1074}\right)^2} - \frac{.7520 \times 16}{16 + 80}\sqrt{1 - \left(\frac{16 + 80}{2 \times .7520 \times 68.1074}\right)^2} \qquad \text{Equ. 6-1}$$

$$= .04860326$$

$$AA = \frac{1.2604}{2}\sqrt{1 - \left(\frac{80}{1.2604 \times 68.1074}\right)^2} - \frac{.752 \times 80}{16 + 80}\sqrt{1 - \left(\frac{16 + 80}{2 \times .7520 \times 68.1074}\right)^2} \qquad \text{Equ. 6-2}$$

$$= .00993254$$

$$CR = (.04860326 + .00993254) \times .3387383 \times 64 \qquad \text{Equ. 6-3}$$

$$= 1.269$$

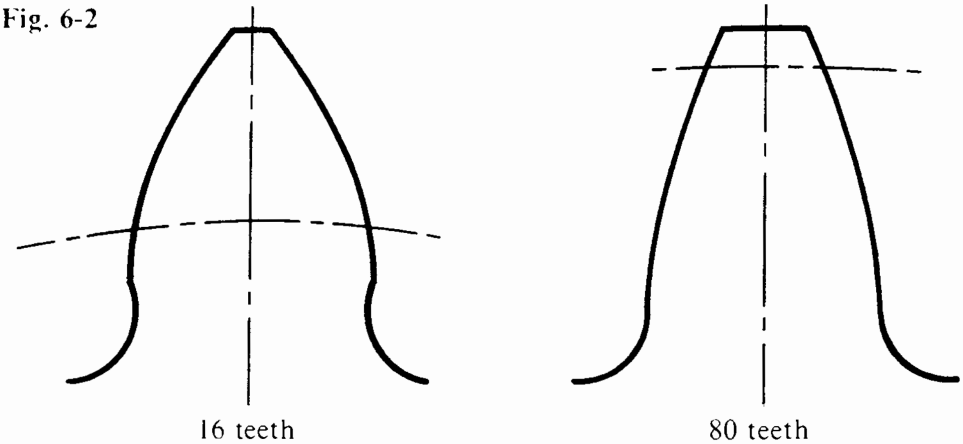
$$RA = \frac{.04860326 \times 100}{.04860326 + .00993254}$$

$$= 83 \text{ percent}$$

The contact ratio has increased to 1.269 and the percentage of recess action to 83, with no added cost.

This example demonstrates graphically the benefits to be derived from the use of longer teeth for fine-pitch instrument gearing.

Fig. 6-2



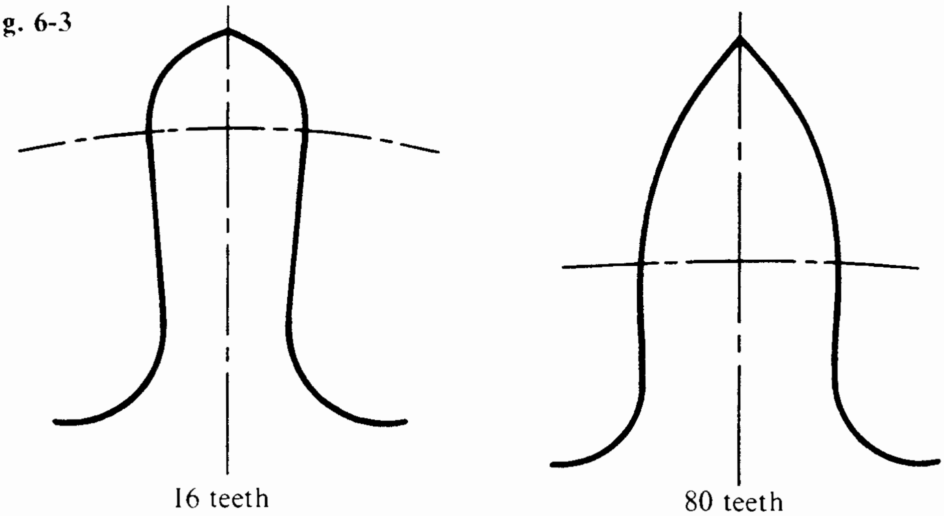
In Fig. 6-2 are shown enlargements of the profiles of the teeth of the pinion and the gear having the PGT-3 tooth form. It will be noted that designing for a high percentage of recess action will also provide teeth of approximately equal strength in the pinion and gear.

Trains of gears in instrument speed-reducing drives consisting of pairs of plastic gears similar to the pinion and gear in Example 6-3 have efficiencies so high that it has occasionally been found necessary to incorporate braking devices to prevent drifting. However, if the drive in such a train were speed-increasing, so that the gears were driving the pinions, the overall efficiency could be so low that the gearing would be self-locking.

For gearing in mechanically driven clockwork mechanisms which have as a power source a slowly falling weight, or unwinding spring, there has evolved over the course of many years a tooth form based on cycloidal, epicycloidal and hypocycloidal curves. For speed-increasing instrument drives cycloidal gears are more efficient than involute gears if the driven pinion has a small number of teeth.

In Fig. 6-3 are shown the enlarged profiles of the teeth of the pinion and gear in Example 6-2 as they would appear if the gear were the driver and the teeth had the cycloidal tooth form.

Fig. 6-3



Cycloidal gears have been molded of the plastics and are being used with success in a number of instrument drives. The manufacture of the mold cavities presents no more problems than does the cavities for involute gears. Because there is no American standard for cycloidal gears, their design is best left to those having some experience in this field.

DRAWING SPECIFICATIONS AND TOLERANCES

The final step in designing a gear is the preparation of a drawing on which are listed the gear data. These data must be so specified that there can be no possiblity of their being misinterpreted. This might appear to be self-evident, but all too often ambiguity in writing gear specifications has resulted in costly and time-consuming changes to expensive molding dies.

Shown in Fig. 7-1 are the data that should appear on the drawing of a spur gear. The data are in a format recommended by the American Gear Manufacturers Association. Some of the information is redundant, but is provided for the convenience of the personnel in manufacturing and inspection departments.

The tooth form is best specified by including on the drawing a dimensioned sketch of the basic rack. The basic racks of the four PGT tooth forms for 1 diametral pitch are shown in Figs. 3-1, 3-2, 3-3 and 3-4. The pressure angle, addendum and whole depth of the basic rack tooth form are also specified in the format under "basic specifications."

When a gear is brought into close mesh with a master gear of known accuracy in a center distance measuring instrument, the testing radius of the gear is the center distance, as thus measured, less half the pitch circle diameter of the master gear. As the gear is rotated with the master through 360°, the center distance and, as a consequence, the testing radius, will vary from a high to a low value. For the gear to be acceptable, the high and low values must be within the maximum and minimum limits specified.

The specifications of the master gear are supplied by the maker, but the master may not have been purchased at the time the gear drawing is being prepared. In that event, the testing radius specified can be the value that would apply if the master were a theoretically perfect gear of known pitch circle diameter, or if the gear were to be rotated in close mesh with a standard rack. Once the master is available, it is preferable then to change the testing radius to conform to the specifications supplied and to specify the master to be used by a tool number.

To determine the testing radius to be specified in the data, the close mesh center distance is calculated by employing Equation 5-1. Two calculations are made: one for the maximum and the other for the minimum calculated circular tooth thickness of the gear. To the maximum center distance is added half the total composite tolerance, and from the minimum is subtracted half the total composite tolerance. The answers give the maximum and minimum values of the close mesh center distance that obtain as the gear is rotated with the master. From each is subtracted half the pitch circle diameter of the master. These answers give the maximum and minimum values of the testing radius.

Additional information about the center distance measuring method of specifying and inspecting gears is to be found in "Accurate Molded Plastic Gears" and the American Gear Manufacturers Association "Gear Handbook."

SPUR & HELICAL GEARS

FIG. 7-1

	SPUR GEAR DATA	SUGGESTED NUMBER OF DECIMAL PLACES
BASIC SPECIFICATIONS	NUMBER OF TEETH	
	DIAMETRAL PITCH	XX.XXXX
	PRESSURE ANGLE	XX°
	STANDARD PITCH DIAMETER	X.XXXX
	TOOTH FORM	
	ADDENDUM	.XXXX
	WHOLE DEPTH	.XXXX
	CALC. CIR. TOOTH THICKNESS ON STD. PITCH CIRCLE	.XXXX MAX. .XXXX MIN.
MANUFACTURING AND INSPECTION	GEAR TESTING RADIUS	X.XXXX MAX. X.XXXX MIN.
	AGMA QUALITY NUMBER	
	MAX. TOTAL COMPOSITE TOLERANCE	.XXXX
	MAX. TOOTH-TO-TOOTH COMPOSITE TOLERANCE	.XXXX
	MASTER GEAR SPECIFICATIONS	
	TESTING PRESSURE (OUNCES)	XX
	DIAMETER OF MEASURING PIN	.XXXX
	MEASUREMENT OVER TWO PINS (FOR SETUP ONLY)	X.XXXX MAX. X.XXXX MIN.
	OUTSIDE DIAMETER	X.XXX $^{+.000}_{-.00X}$
	MAX. ROOT DIAMETER	X.XXX
ENGINEERING REFERENCES	MATING GEAR PART NUMBER	
	NUMBER OF TEETH IN MATING GEAR	
	OPERATING CENTER DISTANCE	X.XXXX MAX. X.XXXX MIN.

Example 7-1

A gear has 37 teeth and a diametral pitch of 20. The PGT-1 tooth form is specified. The calculated circular tooth thickness is .0883 max., .0859 min. The gear is to be to the accuracy required of AGMA Quality No. Q7. Determine the testing radius to be specified on the drawing. Assume that the master gear has 40 teeth and a circular tooth thickness of .0785.

From the AGMA "Gear Handbook" it is found that a gear having 37 teeth, a diametral pitch of 20, and having assigned to it a Quality Number of Q7 has a total composite tolerance of .0040.

(1) Find the close mesh center distance of gear and master for a gear tooth thickness of .0883.

$$C = \frac{(N_1 + N_2) \times .46984631}{P \times \cos \phi_1} \qquad \text{Equ. 5-1}$$

$$N = 37 \qquad N_2 = 40 \qquad P = 20$$

$$\text{inv } \phi_1 = \frac{P(t_1 + t_2) - \pi}{N_1 + N_2} + .01490438$$

$$t_1 = .0883 \qquad t_2 = .0785 \qquad \pi = 3.1415926$$

$$\text{inv } \phi_1 = \frac{20(.0883 + .0785) - 3.1415926}{37 + 40} + .01490438$$

$$= .01742915$$

$$\phi_1 = 21.032704$$

$$\cos \phi_1 = .93337572$$

$$C = \frac{(37 + 40) \times .46984631}{20 \times .93337572} = 1.9380$$

(2) Find close mesh center distance of gear and master for a gear tooth thickness of .0859

$$\text{inv } \phi_1 = \frac{20(.0859 + .0785) - 3.1415926}{37 + 40} + .01490438$$

$$= .01680577$$

$$\phi_1 = 20.788035°$$

$$\cos \phi_1 = .93489981$$

$$C = \frac{(37 + 40) \times .46984631}{20 \times .93489981} = 1.9349$$

(3) Find the testing radius.

$$\text{Pitch circle diameter of master gear} = \frac{40}{20} = 2.0000$$

Total composite tolerance of gear = .0040

$$\text{Maximum testing radius} = 1.9380 + \frac{.0040}{2} - \frac{2.0000}{2} = .9400$$

$$\text{Minimum testing radius} = 1.9349 - \frac{.0040}{2} - \frac{2.0000}{2} = .9329$$

Testing radius = .9400 max. .9329 min.

SPUR & HELICAL GEARS

To obtain accurate measurements of the testing radius, the total composite error and the tooth-to-tooth composite error of a gear, by rotating it with a master in a center distance measuring instrument, the pressure between gear and master is set at a predetermined value which is specified in the gear data as the "testing pressure" in ounces.

In the AGMA "Gear Handbook" are to be found the loads recommended for metal gears, with a further recommendation that half the values given be used for non-metallic gears. These should be regarded as suggested values. Once samples of the gears are available, the testing pressure may be adjusted to a value that provides the most representative readings, but the pressure to be used must be agreed upon and used by the inspection departments of both purchaser and vendor of the gears.

The tooth thickness of a gear may be determined by taking a measurement over pins placed between the teeth. It is not an accurate method, particularly when employed to find the tooth thickness of fine pitch gears molded of the plastics. The proviso in the gear data that the values specified be for "setup only" can be construed to mean that the results obtained may be used as an approximate check of tooth thickness when a gear mold is in the process of being brought on stream.

The equation used to find the measurement over pins of gears having any one of the PGT tooth forms is as follows,

Equation 7-1

(a) Gears having even number of teeth.

$$M = \frac{.93969262 \times N}{P \times \cos \phi_1} + d$$

(b) Gears having odd numbers of teeth.

$$M = \frac{.93969262 \times N}{P \times \cos \phi_1} \left(\cos \frac{90^\circ}{N} \right) + d$$

Where:

M = measurement over two pins
N = number of teeth
P = diametral pitch
d = diameter of measuring pin
ϕ_1 = angle whose involute is $\frac{P}{N}\left[t + (1.0641778 \times d)\right] + .01490438 - \frac{\pi}{N}$
t = circular tooth thickness
π = 3.1415926

Example 7-2

The gear in Example 7-1 has 37 teeth, a diametral pitch of 20, the PGT-1 tooth form and a calculated circular tooth thickness of .0883 max., .0859 min. Find the measurement over two pins.

The gear has an odd number of teeth, therefore Equation 7-1(b) is used.

$$M = \frac{.93969262 \times N}{P \times \cos \phi_1}\left(\cos \frac{90°}{N}\right) + d \qquad \text{Equ. 7-1(b)}$$

$$\text{inv } \phi_1 = \frac{P}{N}\left[t + (1.0641778 \times d)\right] + .01490438 - \frac{\pi}{N}$$

$$N = 37 \quad P = 20 \quad t = .0883 \text{ max. } .0859 \text{ min.} \quad \pi = 3.1415926$$

The diameter of the measuring pin should be such that M is greater than the outside diameter of the gear, but permits the pin to rest on, or near the operating pitch circle. As a guide, pins having a diameter of approximately $\frac{1.728}{P}$ may be tried for gears, and approximately $\frac{1.92}{P}$ for pinions having tooth thicknesses increased over standard.

In this example try a pin having a diameter of:

$$d = \frac{1.728}{P}$$

$$= \frac{1.728}{20}$$

$$= .0864 \text{ (Let } d = .0900)$$

$$\text{(1) inv } \phi_1 = \frac{20}{37}\left[.0883 + (1.0641778 \times .090)\right] + .01490438 - \frac{3.1415926}{37}$$

$$= .02949701$$

$$\phi_1 = 24.873228°$$

$$\cos \phi_1 = .90724065$$

$$M = \frac{.93969262 \times 37}{20 \times 90724065}\left(\cos \frac{90°}{37}\right) + .0900$$

$$= \frac{.93969262 \times 37}{20 \times .90724065}\left(.99909896\right) + .0900 = 2.0044$$

$$\text{(2) inv } \phi_1 = \frac{20}{37}\left[.0859 + (1.0641778 \times .0900)\right] + .01490438 - \frac{3.1415926}{37}$$

$$= .02819971$$

$$\phi_1 = 24.521806°$$

$$\cos \phi_1 = .90980338$$

$$M = \frac{.93969262 \times 37}{20 \times .90980338}\left(.99909896\right) + .0900 = 1.9991$$

Measurement over two .0900 pins = 2.0044 max., 1.9991 min.

SPUR & HELICAL GEARS

The application of tolerances to a gear is part of the design process and should not be undertaken as an afterthought. In the American Gear Manufacturers Association "Gear Handbook" are to be found a list of gear applications and the Quality Number suggested for each application, together with the tolerances recommended for tooth thickness, by Quality Number and diametral pitch. Gears of the plastics can be molded to be within the range of these Quality Numbers and tolerances, but the necessity to specify highly precise, fine-pitch gearing for instrument and similar applications can frequently be avoided by use of one of the longer teeth of the PGT System, as demonstrated in Example 6-3.

As with any other manufacturing process, the closer the tolerances required of a molded gear the more costly will be the part. Production runs of multi-cavity gear molds can last for many weeks of 24 hours per day operation. During the course of the run there will occur minor variations in the temperature and pressure of the plastic, and the temperature of the mold. Chemical engineers also work to tolerances, so there will be variations in the molecular weight of the plastic from batch to batch. All these variations will result in minor changes to the sizes of the gears. To give the molder as much latitude as possible, the gear tolerances should be as generous as the application will permit. The best possible gear for any application is the least expensive gear that will do the job required of it.

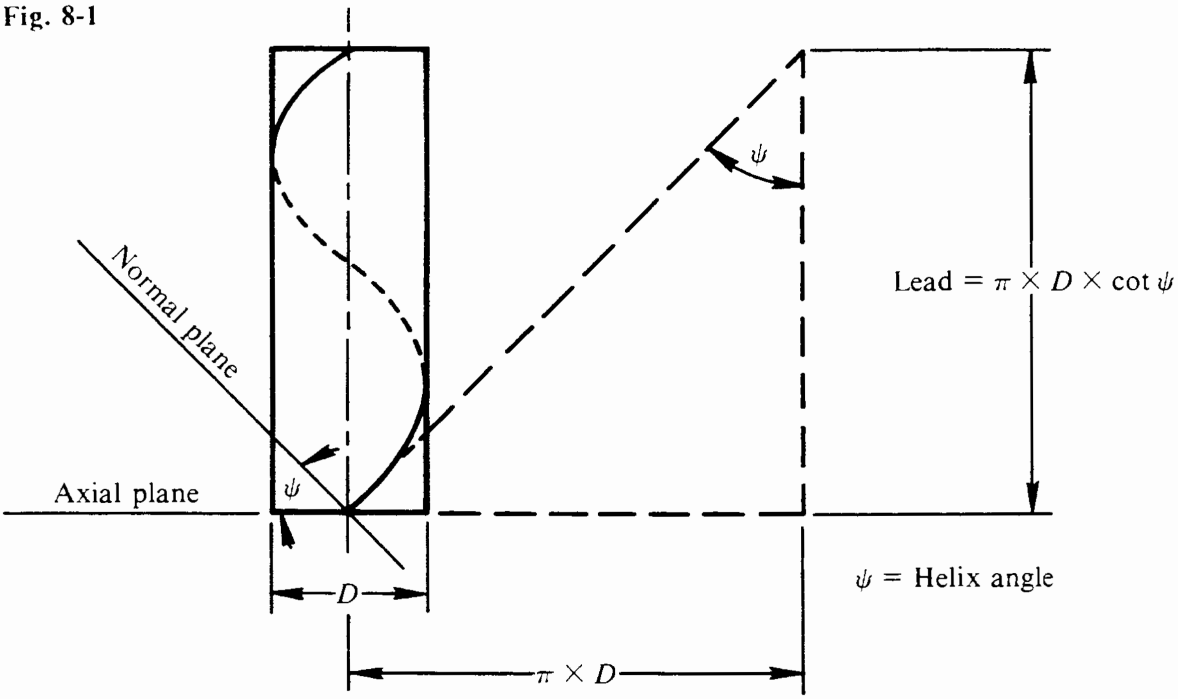
To hold down the piece price of a molded gear, not only must the tolerances be as generous as is permissable, but the molding die must be a highly precise tool. To have and to hold that precision, it must be fully hardened and made to gauge tolerances, and the cavities must be held to the accuracies demanded of a master gear. It would be patently absurd to expect Quality Q7 gears from Quality Q7 cavities. Any relaxation in the quality and precision of gear molding dies will result in a need for a closer control of the molding process, too much down time and a high percentage of rejects: all factor contributing to higher than necessary production costs. Low cost accurate molded plastic gears and cheap tooling are just not compatible.

HELICAL GEARS

A helical gear differs from a spur gear in that the teeth, instead of being parallel to the axis of the shaft on which the gear is mounted, are formed on a spiral that winds around the axis. Included in the data of a helical gear are three pieces of information additional to what appears in the data of a spur gear. These are the helix angle, the direction of the helix—either right hand or left hand—and the lead. Mating helical gears on parallel shafts have the same helix angle but opposite hands of helix.

The lead of a helical gear is the distance a point on the flank of a tooth would travel along the axis in moving around the axis through 360°. From Fig. 8-1 it will be seen that the lead is equal to the circumference of a circle containing the point multiplied by the cotangent of the helix angle.

Fig. 8-1



The helix angle of a helical gear is always specified as being the helix angle at the standard pitch circle. The lead is calculated by use of Equation 8-1.

Equation 8-1

$$L = \pi \times D \times \cot \psi$$

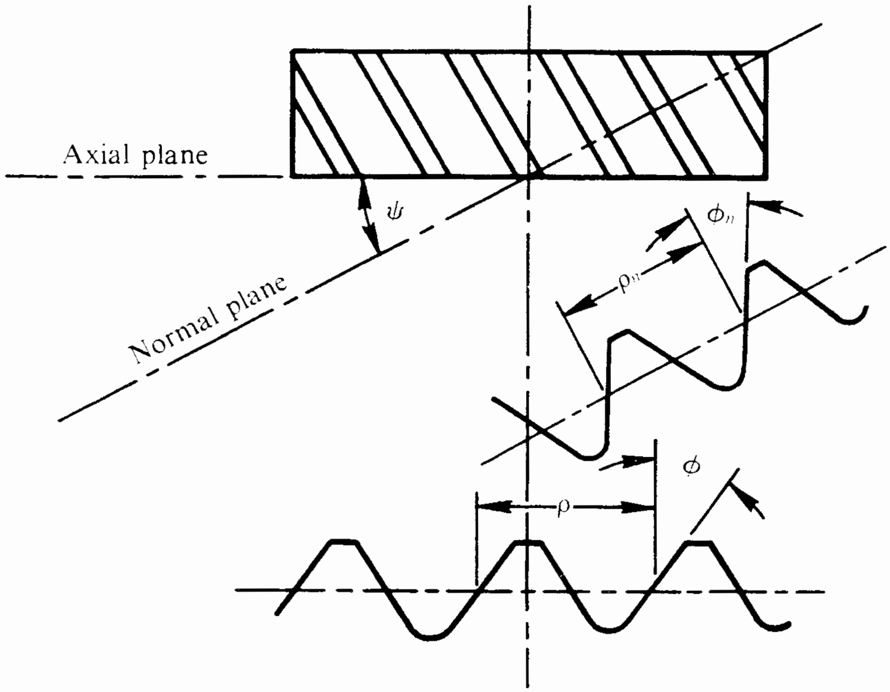
Where:

L = lead
π = 3.1415926
D = standard pitch diameter
ψ = helix angle

SPUR & HELICAL GEARS

It can be said of a helical gear that it has two basic racks, one in the normal plane and the other in the axial plane. Fig. 8-2 illustrates the two racks and their relationship to one another.

Fig. 8-2



ψ = helix angle

ρ = circular pitch ρ_n = normal circular pitch $\rho = \dfrac{\rho_n}{\cos\psi}$

ϕ = pressure angle ϕ_n = normal pressure angle $\phi = \tan^{-1}\left(\dfrac{\tan\phi_n}{\cos\psi}\right)$

$P = P_n \times \cos\psi$ where: P_n = normal diametral pitch
P = diametral pitch

$$D = \frac{N}{P} = \frac{N}{P_n \times \cos\psi}$$

where D = standard pitch diameter
N = number of teeth

The tooth form of a helical gear is specified as being the tooth form conforming to the basic rack in the normal place. For all helical gears having helix angles of not more than 23°, and operating on parallel shafts, the tooth form specified is the same as that for spur gears. This is not obligatory, but it permits the use of spur gear hobs to generate the teeth in helical gears. While hobs are not used to make gears molded of the plastics, there is no good reason to depart from what has become an accepted practice in the world of gear engineering.

As with spur gears, a pair of mating helical gears must have an adequate contact ratio. But helical gears have a helical overlap which, in effect, is an additional contact ratio.

The axial pitch of a helical gear is the lead divided by the number of teeth.

$$\rho_x = \frac{L}{N}$$

Where: ρ_x = axial pitch L = lead N = number of teeth

The axial pitch is also given by the equation

$$\rho_x = \frac{\pi}{P_n \times \sin\psi}$$

Where: $\pi = 3.1415926$ P_n = normal diametral pitch

ψ = helix angle

A helical gear having a face width equal to its axial pitch has a helical overlap of 1.0. For any real benefit to accrue from the use of helical gears the helical overlap should be 2.0. In other words, the face width should be twice the axial pitch, or as close to that dimension as other design considerations permit. It is this helical overlap, or face contact ratio, that makes helical gears superior to spur gears for drives requiring the maximum in smooth, quiet operation.

Unlike spur gears, helical gears exert end thrusts on the bearings carrying the shafts on which the gears are mounted. The amount of end thrust increases with the helix angle, and high helix angles require special tooth forms. The helix angle of gears molded of the plastics should be kept between 13° and 23°. Given a choice it is recommended that a helix angle of 18° be specified. No erudite explanation can be offered for this recommendation, but from experience it seems that helical gears having a helix angle of 18° invariably perform well.

All that has been written about spur gears in the preceding chapters holds true for helical gears. The PGT-1 tooth form is usually specified for helical gears, although there may be occasions when the longer tooth of the PGT-2 tooth form can be used to advantage. The equations employed in the design of helical gears are given in the chapter that follows. They are the spur gear equations modified to account for the helix. They could, in fact, be used to design spur gears by giving the helix angle a value of zero.

On the drawing of a helical gear the data are presented in the format shown in Fig. 8-3.

SPUR & HELICAL GEARS

FIG. 8-3

	HELICAL GEAR DATA	SUGGESTED NUMBER OF DECIMAL PLACES
BASIC SPECIFICATIONS	NUMBER OF TEETH	
	NORMAL DIAMETRAL PITCH	XX.XXXX
	NORMAL PRESSURE ANGLE	XX°
	HELIX ANGLE	XX.XXXX°
	HAND OF HELIX	L.H. or R.H.
	STANDARD PITCH DIAMETER	X.XXXX
	TOOTH FORM	
	ADDENDUM	.XXXX
	WHOLE DEPTH	.XXXX
	CALC. NORMAL CIR. TOOTH THICKNESS ON STD. PITCH CIRCLE	.XXXX MAX. .XXXX MIN.
MANUFACTURING AND INSPECTION	GEAR TESTING RADIUS	X.XXXX MAX. X.XXXX MIN.
	AGMA QUALITY NUMBER	
	MAXIMUM TOTAL COMPOSITE TOLERANCE	.XXXX
	MAX. TOOTH-TO-TOOTH COMPOSITE TOLERANCE	.XXXX
	MASTER GEAR SPECIFICATIONS	
	TESTING PRESSURE (OUNCES)	XX
	DIAMETER OF MEASURING PIN	.XXXX
	MEASUREMENT OVER TWO PINS (FOR SETUP ONLY)	X.XXXX MAX. X.XXXX MIN.
	LEAD	
	OUTSIDE DIAMETER	X.XXX $^{+.000}_{-.00X}$
	MAX. ROOT DIAMETER	X.XXXX
ENGINEERING REFERENCES	MATING GEAR PART NUMBER	
	NUMBER OF TEETH IN MATING GEAR	
	OPERATING CENTER DISTANCE	X.XXXX MAX. X.XXXX MIN.

THE HELICAL GEAR EQUATIONS

Given the normal circular tooth thickness of a helical gear, the outside and root diameters are obtained by using the appropriate equation in Tables 9-1 and 9-2.

Table 9-1

Tooth form	Outside diameter	**Equ. No.**
PGT-1	$D_o = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 2.3158\right) + \left(2.7475 \times t_n\right)$	**9-1**
PGT-2	$D_o = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 2.0158\right) + \left(2.7475 \times t_n\right)$	**9-2**

Table 9-2

Tooth form	Root diameter	**Equ. No.**
PGT-1	$D_R = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 6.9758\right) + \left(2.7475 \times t_n\right)$	**9-3**
PGT-2	$D_R = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 7.2758\right) + \left(2.7475 \times t_n\right)$	**9-4**

Where:
- D_o = outside diameter
- D_R = root diameter
- P_n = normal diametral pitch
- N = number of teeth
- ψ = helix angle
- t_n = normal cir. tooth thickness on std. pitch circle

Example 9-1

A helical gear has 40 teeth, a normal diametral pitch of 32, a helix angle of 18°, the PGT-1 tooth form and a normal circular tooth thickness of .0475. Find the outside and root diameters.

$$D_o = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 2.3158\right) + \left(2.7475 \times t_n\right) \qquad \text{Equ. 9-1}$$

$P_n = 32 \quad N = 40 \quad \psi = 18° \quad t_n = .0475$

$$D_o = \frac{1}{32}\left(\frac{40}{.95105652} - 2.3158\right) + \left(2.7475 \times .0475\right) = 1.3725$$

$$D_R = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 6.9758\right) + \left(2.7475 \times .0475\right) \qquad \text{Equ. 9-3}$$

$$= \frac{1}{32}\left(\frac{40}{.95105652} - 6.9758\right) + \left(2.7475 \times .0475\right) = 1.2268$$

SPUR & HELICAL GEARS

Given the normal circular tooth thickness of a helical pinion having a small number of teeth, the maximum outside diameter which will still provide an adequate top land is obtained by using Equation 9-5.

Equation 9-5

$$D_o(\max) = \frac{N \times \cos\phi}{P_n \times \cos\psi \times 1.017 \times \cos\phi_1}$$

where:

$D_o(\max)$ = maximum outside diameter

ψ = helix angle

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right)$$

$$\text{inv}\,\phi_1 = \frac{t_n \times P_n}{N} + \text{inv}\,\phi$$

t_n = normal circular tooth thickness

Example 9-2

A helical pinion has 8 teeth, a normal diametral pitch of 32, a helix angle of 18° and a normal circular tooth thickness of .0666. Determine the maximum outside diameter.

$$D_o(\max) = \frac{N \times \cos\phi}{P_n \times \cos\psi \times 1.017 \times \cos\phi_1} \qquad \text{Equ. 9-5}$$

$N = 8 \quad P_n = 32 \quad \psi = 18° \quad \cos\psi = .95105652$

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right) = \tan^{-1}\left(\frac{.36397023}{.95105652}\right) = 20.941896°$$

$\cos\phi = .93394337 \quad \text{inv}\,\phi = .01719592$

$$\text{inv}\,\phi_1 = \frac{t_n \times P_n}{N} + \text{inv}\,\phi$$

$$= \frac{.0666 \times 32}{8} + .01719592 = .2835959$$

$\phi_1 = 48.490408°$

$\cos\phi_1 = .66274543$

$$D_o(\max) = \frac{8 \times .93394337}{32 \times .95105652 \times 1.017 \times .66274543} = .3642$$

Find outside diameter given by Equation 9-1

$$D_o = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 2.3158\right) + \left(2.7475 \times t_n\right) \qquad \text{Equ. 9-1}$$

$$= \frac{1}{32}\left(\frac{8}{.95105652} - 2.3158\right) + \left(2.7475 \times .0666\right) = .3735$$

The outside diameter to be specified is .3642, the lesser of the answers given by Equations 9-5 and 9-1.

Given the number of teeth, the normal diametral pitch, the helix angle and the tooth form of a helical gear, the minimum normal circular tooth thickness required to avoid objectionable undercut is determined by using the appropriate equation in Table 9-3.

Table 9-3

Tooth form	Minimum normal circular tooth thickness	Equ. No.
PGT-1	$t_n = \frac{1}{P_n}\left(2.3329 - \frac{N \times (1 - \cos^2 \phi)}{2.7475 \times \cos \psi}\right)$	**9-6**
PGT-2	$t_n = \frac{1}{P_n}\left(2.4793 - \frac{N \times (1 - \cos^2 \phi)}{2.7475 \times \cos \psi}\right)$	**9-7**

where:

t_n = minimum normal circular tooth thickness
P_n = normal diametral pitch
N = number of teeth
ψ = helix angle
$\phi = \tan^{-1}\left(\frac{.36397023}{\cos \psi}\right)$

Example 9-3

A helical gear has 12 teeth, a normal diametral pitch of 32, a helix angle of 18° and the PGT-1 tooth form. Determine the minimum normal circular tooth thickness required to avoid objectionable undercutting.

$$t_n = \frac{1}{P_n}\left(2.3329 - \frac{N(1 - \cos^2 \phi)}{2.7475 \times \cos \psi}\right) \qquad \text{Equ. 9-6}$$

$P_n = 32 \qquad N = 12 \qquad \psi = 18° \qquad \cos \psi = .95105652$

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos \psi}\right)$$

$$= \tan^{-1}\left(\frac{.36397023}{.95105652}\right)$$

$$= 20.941896°$$

$$\cos \phi = .93394337$$

$$\cos^2 \phi = .87225022$$

$$t_n = \frac{1}{32}\left(2.3329 - \frac{12 \times (1 - .87225022)}{2.7475 \times .95105652}\right)$$

$$= .0546$$

Given the numbers of teeth, the normal diametral pitch and the helix angle of a pair of mating helical gears having the PGT-1 tooth form, the normal circular tooth thicknesses for balanced tooth strength are obtained by use of Equations Nos. 9-8, 9-9, 9-10.

Equation 9-8

Pinion and gear both having less teeth than given by

$$N = \frac{2.0938 \times \cos\psi}{1 - \cos\phi}$$

$$t_{n1} = \frac{1}{P_n}\left[2.3329 - \frac{.36397023 \times N_1 \times (1 - \cos\phi)}{\cos\psi}\right] \text{———} (a)$$

$$t_{n2} = \frac{1}{P_n}\left[2.3329 - \frac{.36397023 \times N_2 \times (1 - \cos\phi)}{\cos\psi}\right] \text{———} (b)$$

Equation 9-9

Pinion having less teeth and gear having more teeth than given by

$$N = \frac{2.0938 \times \cos\psi}{1 - \cos\phi}$$

$$t_{n1} = \frac{1}{P_n}\left[2.3329 - \frac{.36397023 \times N_1 \times (1 - \cos\phi)}{\cos\psi}\right] \text{———} (a)$$

$$t_{n2} = \frac{N_2}{P_n}\left[\frac{P_n \times B \times \cos\psi}{N_2 - (2.0938 \times \cos\psi)} + \operatorname{inv}\phi_2 - \operatorname{inv}\phi\right] \text{———} (b)$$

Equation 9-10

Pinion and gear both having more teeth than given by

$$N = \frac{2.0938 \times \cos\psi}{1 - \cos\phi}$$

$$t_{n1} = \left[\frac{N_1 \times (N_2 - (2.0938 \times \cos\psi))}{N_1 - (2.0938 \times \cos\psi)}\right]\left[\frac{t_{n2}}{N_2} + \frac{\operatorname{inv}\phi - \operatorname{inv}\phi_2}{P_n}\right] - N_1\left[\frac{\operatorname{inv}\phi - \operatorname{inv}\phi_1}{P_n}\right]$$

where:

N = number of teeth determining which equation to use

ψ = helix angle

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right)$$

N_1 = number of teeth in pinion

N_2 = number of teeth in gear

t_{n1} = normal circular tooth thickness of pinion

t_{n2} = normal circular tooth thickness of gear

$$\phi_1 = \cos^{-1}\left[\frac{N_1 \times \cos\phi}{N_1 - (2.0938 \times \cos\psi)}\right]$$

$$\phi_2 = \cos^{-1}\left[\frac{N_2 \times \cos\phi}{N_2 - (2.0938 \times \cos\psi)}\right]$$

$$B = \frac{N_1 \times \cos\phi}{P_n \times \cos\psi}\left(\frac{P_n \times t_{n1}}{N_1} + \operatorname{inv}\phi\right)$$

Example 9-5

The pinion in the previous example drives a gear having 35 teeth. Determine the normal circular tooth thicknesses for balanced tooth strength.

$N = 30.146$. The pinion has less than 30.146 teeth, the gear has more. Use Equation 9-9

$$t_{n1} = \frac{1}{P_n}\left[2.3329 - \frac{.36397023 \times N_1 \times (1 - \cos\phi)}{\cos\psi}\right] \qquad \text{Equ. 9-9(a)}$$

$P_n = 24 \quad N_1 = 12 \quad \cos\phi = .93394337 \quad \cos\psi = .95105652$

$$t_{n1} = \frac{1}{24}\left[2.3329 - \frac{.36397023 \times 12 \times (1 - .93394337)}{.95105652}\right]$$

$$= .0846$$

$$t_{n2} = \frac{N_2}{P_n}\left[\frac{P_n \times B \times \cos\psi}{N_2 - (2.0938 \times \cos\psi)} + \text{inv}\,\phi_2 - \text{inv}\,\phi\right] \qquad \text{Equ. 9-9(b)}$$

$N_2 = 35 \quad P_n = 24 \quad \cos\psi = .95105652$

$\phi = 20.941896° \quad \text{inv}\,\phi = .01719592$

$$\phi_2 = \cos^{-1}\left[\frac{N_2 \times \cos\phi}{N_2 - (2.0938 \times \cos\psi)}\right]$$

$$= \cos^{-1}\left[\frac{35 \times .93394337}{35 - (2.0938 \times .95105652)}\right]$$

$$= \cos^{-1}\,(.99028558)$$

$$= 7.992785°$$

$\text{inv}\,\phi_2 = .00091201$

$$B = \frac{N_1 \times \cos\phi}{P_n \times \cos\psi}\left(\frac{P_n \times t_{n1}}{N_1} + \text{inv}\,\phi\right)$$

$N_1 = 12 \quad \cos\phi = .93394337 \quad P_n = 24$

$t_n = .0846 \quad \text{inv}\,\phi = .01719592 \quad \cos\psi = .95105652$

$$B = \frac{12 \times .93394337}{24 \times .95105652}\left(\frac{24 \times .0846}{12} + .01719592\right)$$

$$= .09152097$$

$$t_{n2} = \frac{35}{24}\left[\frac{24 \times .09152097 \times .95105652}{35 - (2.0938 \times .95105652)} + .00091201 - .01719592\right]$$

$$= .0685$$

Normal circular tooth thickness of pinion = .0846

Normal circular tooth thickness of gear = .0685

Example 9-4

A helical pinion having 12 teeth drives a gear having 23 teeth. The normal diametral pitch is 24 and the helix angle is 18°. The PGT-1 tooth form is specified. Determine the normal circular tooth thicknesses for balanced tooth strength.

$$N = \frac{2.0938 \times \cos\psi}{1 - \cos\phi}$$

$$\psi = 18^\circ \qquad \cos\psi = .95105652$$

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right) = \tan^{-1}\left(\frac{.36397023}{.95105652}\right) = 20.941896^\circ$$

$$\cos\phi = .93394337$$

$$N = \frac{2.0938 \times .95105652}{1 - .93394337}$$

$$= 30.146$$

Both pinion and gear in this example have less than 30.146 teeth. Use Equation 9-8.

$$t_{n1} = \frac{1}{P_n}\left[2.3329 - \frac{.36397023 \times N_1 \times (1 - \cos\phi)}{\cos\psi}\right] \qquad \text{Equ. 9-8(a)}$$

$$P_n = 24 \qquad N_1 = 12 \qquad \cos\phi = .93394337 \qquad \cos\psi = .95105652$$

$$t_{n1} = \frac{1}{24}\left[2.3329 - \frac{.36397023 \times 12 \times (1 - .93394337)}{.95105652}\right]$$

$$= .0846$$

$$t_{n2} = \frac{1}{P_n}\left[2.3329 - \frac{.36397023 \times N_2 \times (1 - \cos\phi)}{\cos\psi}\right] \qquad \text{Equ. 9-8(b)}$$

$$N_2 = 23$$

$$= \frac{1}{24}\left[2.3329 - \frac{.36397023 \times 23 \times (1 - .93394337)}{.95105652}\right]$$

$$= .0730$$

Normal circular tooth thickness of pinion = .0846

Normal circular tooth thickness of gear = .0730

Example 9-5

The pinion in the previous example drives a gear having 35 teeth. Determine the normal circular tooth thicknesses for balanced tooth strength.

$N = 30.146$. The pinion has less than 30.146 teeth, the gear has more. Use Equation 9-9

$$t_{n1} = \frac{1}{P_n}\left[2.3329 - \frac{.36397023 \times N_1 \times (1 - \cos\phi)}{\cos\psi}\right] \qquad \text{Equ. 9-9(a)}$$

$P_n = 24 \quad N_1 = 12 \quad \cos\phi = .93394337 \quad \cos\psi = .95105652$

$$t_{n1} = \frac{1}{24}\left[2.3329 - \frac{.36397023 \times 12 \times (1 - .93394337)}{.95105652}\right]$$

$$= .0846$$

$$t_{n2} = \frac{N_2}{P_n}\left[\frac{P_n \times B \times \cos\psi}{N_2 - (2.0938 \times \cos\psi)} + \text{inv}\,\phi_2 - \text{inv}\,\phi\right] \qquad \text{Equ. 9-9(b)}$$

$N_2 = 35 \quad P_n = 24 \quad \cos\psi = .95105652$

$\phi = 20.941896° \quad \text{inv}\,\phi = .01719592$

$$\phi_2 = \cos^{-1}\left[\frac{N_2 \times \cos\phi}{N_2 - (2.0938 \times \cos\psi)}\right]$$

$$= \cos^{-1}\left[\frac{35 \times .93394337}{35 - (2.0938 \times .95105652)}\right]$$

$$= \cos^{-1}(.99028558)$$

$$= 7.992785°$$

$\text{inv}\,\phi_2 = .00091201$

$$B = \frac{N_1 \times \cos\phi}{P_n \times \cos\psi}\left(\frac{P_n \times t_{n1}}{N_1} + \text{inv}\,\phi\right)$$

$N_1 = 12 \quad \cos\phi = .93394337 \quad P_n = 24$

$t_n = .0846 \quad \text{inv}\,\phi = .01719592 \quad \cos\psi = .95105652$

$$B = \frac{12 \times .93394337}{24 \times .95105652}\left(\frac{24 \times .0846}{12} + .01719592\right)$$

$$= .09152097$$

$$t_{n2} = \frac{35}{24}\left[\frac{24 \times .09152097 \times .95105652}{35 - (2.0938 \times .95105652)} + .00091201 - .01719592\right]$$

$$= .0685$$

Normal circular tooth thickness of pinion = .0846

Normal circular tooth thickness of gear = .0685

Example 9-6

The gear having 35 teeth in the previous example becomes the pinion and drives a gear having 71 teeth. Determine the normal circular tooth thicknesses for balanced tooth strength.

$N = 30.146$. Both pinion and gear have more than 30.146 teeth. Use Equation 9-10.

$$t_{n1} = \left[\frac{N_1 \times (N_2 - (2.0938 \times \cos\psi))}{N_1 - (2.0938 \times \cos\psi)}\right]\left[\frac{t_{n2}}{N_2} + \frac{\text{inv}\,\phi - \text{inv}\,\phi_2}{P_n}\right] - N_1\left[\frac{\text{inv}\,\phi - \text{inv}\,\phi_1}{P_n}\right] \text{Equ. 9-10}$$

$N_1 = 35 \quad N_2 = 71 \quad \cos\psi = .95105652 \quad P_n = 24 \quad \cos\phi = .93394337$

$$\phi_1 = \cos^{-1}\left(\frac{N_1 \times \cos\phi}{N_1 - (2.0938 \times \cos\psi)}\right)$$

$$= \cos^{-1}\left(\frac{35 \times .93394337}{35 - (2.0938 \times .95105652)}\right)$$

$$= \cos^{-1}\ (.99028558)$$

$$= 7.992785°$$

$$\text{inv}\,\phi_1 = .00091201$$

$$\phi_2 = \cos^{-1}\left(\frac{N_2 \times \cos\phi}{N_2 - (2.0938 \times \cos\psi)}\right)$$

$$= \cos^{-1}\left(\frac{71 \times .93394337}{71 - (2.0938 \times .95105652)}\right)$$

$$= \cos^{-1}\ (.96089334)$$

$$= 16.076391°$$

$$\text{inv}\,\phi_2 = .00760289$$

Let t_{n2} = standard normal circular tooth thickness

$$= \frac{\pi}{2P_n}$$

$$= \frac{3.1415926}{2 \times 24}$$

$$= .0654$$

$$t_{n1} = \left[\frac{35 \times (71 - (2.0938 \times .95105652))}{35 - (2.0938 \times .95105652)}\right]\left[\frac{.0654}{71} + \frac{.01719592 - .00760289}{24}\right] - 35\left[\frac{.01719592 - .0009120}{24}\right]$$

$$= .0729$$

Normal circular tooth thickness of pinion = .0729

Normal circular tooth thickness of gear = .0654

Given the numbers of teeth, the normal diametral pitch, and the helix angle of a pair of mating helical gears having any one of the PGT tooth forms, the center distance at which the gears are in close mesh is determined by use of Equation 9-11.

Equation 9-11

$$C = \frac{(N_1 + N_2) \times \cos\phi}{2 \times P_n \times \cos\psi \times \cos\phi_1}$$

where:

C = close mesh center distance
N_1 = number of teeth in first gear
N_2 = number of teeth in second gear
P_n = normal diametral pitch
ψ = helix angle
$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right)$
ϕ_1 = angle whose involute is $\frac{P_n(t_{n1} + t_{n2}) - \pi}{N_1 + N_2} + \text{inv}\,\phi$

where:

t_{n1} = normal circular tooth thickness of 1st gear
t_{n2} = normal circular tooth thickness of 2nd gear
$\pi = 3.1415926$

Example 9-7

Two mating helical gears have the basic specifications given below. Find the center distance at which they will be in close mesh.

	1st gear	2nd gear
Number of teeth	12	36
Normal diametral pitch	32	32
Helix angle	18°	18°
Tooth form	PGT-1	PGT-1
Calc. normal cir. tooth thickness on std. pitch circle	.0546	.0491

$$C = \frac{(N_1 + N_2) \times \cos\phi}{2 \times P_n \times \cos\psi \times \cos\phi_1} \qquad \text{Equ. 9-11}$$

$N_1 = 12 \quad N_2 = 36 \quad P_n = 32 \quad \psi = 18° \quad \cos\psi = .95105652$

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right) = \tan^{-1}\left(\frac{.36397023}{.95105652}\right) = 20.941896° \quad \cos\phi = .93394337$$

$\text{inv}\,\phi = .01719592$

$$\text{inv}\,\phi_1 = \frac{P_n(t_{n1} + t_{n2}) - \pi}{N_1 + N_2} + \text{inv}\,\phi$$

$t_{n1} = .0546 \quad t_{n2} = .0491 \quad \pi = 3.1415926$

$$\text{inv}\,\phi_1 = \frac{32(.0546 + .0491) - 3.1415926}{12 + 36} + .01719592 = .02087941$$

Example 9-8

Listed below are the basic specifications of a pair of mating helical gears. The first gear is molded of a nylon and the second of an acetal. The housing is an aluminum die casting. The maximum temperature to which the gears will be subjected is 170° F. The gears might also be required to operate in an environment having a high relative humidity for extended periods. The bearings in the housing have a maximum allowable runout of .0005 *TIR*. The gears are required to have an accuracy corresponding to AGMA Quality Number Q7. Determine the minimum operating center distance.

	1st gear	2nd Gear
Number of teeth	15	45
Normal diametral pitch	48	48
Helix angle	18.600°	18.600°
Tooth form	PGT-1	PGT-1
Calc. normal cir. tooth thickness on std. pitch circle	.0388	.0327

According to the AGMA "Gear Handbook" the 1st gear has a maximum total composite tolerance of .0026 and the 2nd gear a maximum total composite tolerance of .0027.

The nylon and acetal of which the gears are molded have coefficients of linear thermal expansion of 4.0×10^{-5} in/in/° F and 4.5×10^{-5} in/in/° F respectively.

The growth of the nylon with moisture absorption could amount to .001 in/in., and the growth of the acetal to .0002 in/in.

The aluminum of which the housing is made has a coefficient of linear thermal expansion of 1.0×10^{-5} in/in/° F.

(1) Determine the center distance at which the gears would be in close mesh.

$$C = \frac{(N_1 + N_2) \times \cos\phi}{2 \times P_n \times \cos\psi \times \cos\phi_1} \qquad \text{Equ. 9-11}$$

$N_1 = 15 \quad N_2 = 45 \quad \psi = 18.600° \quad \cos\psi = .94776841 \quad P_n = 48$

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right) = \tan^{-1}\left(\frac{.36397023}{.94776841}\right) = 21.008221°$$

$\cos\phi = .93352900 \quad \text{inv}\,\phi = .01736604$

$$\text{inv}\,\phi_1 = \frac{P_n(t_{n1} + t_{n2}) - \pi}{N_1 + N_2} + \text{inv}\,\phi$$

$t_{n1} = .0388 \quad t_{n2} = .0327 \quad \pi = 3.1415926$

$$\text{inv}\,\phi_1 = \frac{48(.0388 + .0327) - 3.1415926}{15 + 45} + .01736604$$

$$= .02220616$$

$\phi_1 = 22.728393° \quad \cos\phi_1 = .92234674$

$$C = \frac{(15 + 45) \times .93352900}{2 \times 48 \times .94776841 \times .92234674} = .6674$$

(Example 9-7 continued)

$\phi_1 = 22.285664 \qquad \cos\phi_1 = .92530463$

$$C = \frac{(12 + 36) \times .93394337}{2 \times 32 \times .95105652 \times .92530463} = .7960$$

Close mesh center distance = .7960

Given the center distance at which a pair of mating helical gears are in close mesh, the amount the minimum operating center distance must exceed the close mesh center distance is determined by use of Equation 9-12.

Equation 9-12

$$\Delta_C = \frac{TCT_1 + TCT_2}{2} + C\left[(T-70)\left(\frac{COEF_1 \times N_1}{N_1 + N_2} + \frac{COEF_2 \times N_2}{N_1 + N_2} - COEF_H\right) + \left(\frac{M_1 \times N_1}{N_1 + N_2} + \frac{M_2 \times N_2}{N_1 + N_2} - M_H\right)\right] + \frac{TIR_1 + TIR_2}{2}$$

where:

Δ_C = required increase in center distance
TCT_1 = maximum total composite tolerance of 1st gear
TCT_2 = maximum total composite tolerance of 2nd gear
C = close mesh center distance
T = maximum temperature to which gears will be subjected (° F)
$COEF_1$ = coefficient of linear thermal expansion of material of 1st gear (in/in/° F)
$COEF_2$ = coefficient of linear thermal expansion of material of 2nd gear (in/in/° F)
$COEF_H$ = coefficient of linear thermal expansion of material of housing (in/in/° F)
N_1 = number of teeth in first gear
N_2 = number of teeth in second gear
M_1 = expansion due to moisture pick-up of material of 1st gear (in/in)
M_2 = expansion due to moisture pick-up of material of 2nd gear (in/in)
M_H = expansion due to moisture pick-up of material of the housing (in/in)
TIR_1 = maximum allowable runout of bearings of first gear
TIR_2 = maximum allowable runout of bearings of second gear

Given the center distance at which a pair of helical gears are in close mesh, the sum of the normal circular tooth thicknesses of the gears is determined by use of Equation 9-13.

Equation 9-13

$$t_{n1} + t_{n2} = \frac{(N_1 + N_2)\,(\text{involute}\,\phi_1 - \text{involute}\,\phi) + \pi}{P_n}$$

where:

t_{n1} = normal circular tooth thickness of 1st gear
t_{n2} = normal circular tooth thickness of 2nd gear
N_1 = number of teeth in 1st gear
N_2 = number of teeth in 2nd gear
$\pi = 3.1415926$
$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right)$
ψ = helix angle
$\phi_1 = \cos^{-1}\left[\frac{(N_1 + N_2) \times \cos\phi}{2 \times P_n \times \cos\psi \times C}\right]$

where: C = close mesh center distance

(2) Determine Δ_C

$$\Delta_C = \frac{TCT_1 + TCT_2}{2} + C\left[(T-70)\left(\frac{COEF_1 \times N_1}{N_1 + N_2} + \frac{COEF_2 \times N_2}{N_1 + N_2} - COEF_H\right) + \left(\frac{M_1 \times N_1}{N_1 + N_2} + \frac{M_2 \times N_2}{N_1 + N_2} - M_H\right)\right] + \frac{TIR_1 + TIR_2}{2}$$

Equ. 9-12

$TCT_1 = .0026 \quad TCT_2 = .0027 \quad C = .6674 \quad T = 170$

$COEF_1 = .00004 \quad COEF_2 = .000045 \quad COEF_H = .00001$

$N_1 = 15 \quad N_2 = 45 \quad M_1 = .001 \quad M_2 = .0002 \quad M_H = .0000 \quad TIR_1 = .0005 \quad TIR_2 = .0005$

$$\Delta_C = \frac{.0026 + .0027}{2} + .6674\left[(170-70)\left(\frac{.00004 \times 15}{15 + 45} + \frac{.000045 \times 45}{15 + 45} - .00001\right) + \left(\frac{.001 \times 15}{15 + 45} + \frac{.0002 \times 45}{15 + 45} - .0000\right)\right] + \frac{.0005 + .0005}{2}$$

$= .0057$

(3) Minimum operating center distance $= C + \Delta_C$

$= .6674 + .0057$

$= .6731$

(4) As an alternative the minimum operating center distance could be established at .6674 and the teeth reduced in thickness. The reduction required is determined by use of Equation 9-13. The value given to "C" in this equation is

$.6674 - \Delta_C$

$= .6674 - .0057$

$= .6617$

(5) Determine the sum of the normal circular tooth thickness for a close mesh center distance of .6617.

$$t_{n1} + t_{n2} = \frac{(N_1 + N_2)(\text{inv}\,\phi_1 - \text{inv}\,\phi) + \pi}{P_n} \qquad \text{Equ. 9-13}$$

$N_1 = 15 \quad N_2 = 45 \quad P_n = 48 \quad \pi = 3.1415926$

$\psi = 18.600° \quad \cos\psi = .94776841$

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right) = \tan^{-1}\left(\frac{.36397023}{.94776841}\right) = 21.008221° \quad \text{inv}\,\phi = .01736604$$

$$\phi_1 = \cos^{-1}\left[\frac{(N_1 + N_2) \times \cos\phi}{2 \times P_n \times \cos\psi \times C}\right]$$

$\cos\phi = \cos 21.008221° = .93352900 \quad C = .6617$

$$\phi_1 = \cos^{-1}\left[\frac{(15 + 45) \times .93352900}{2 \times 48 \times .94776841 \times .6617}\right]$$

$$= 21.5111950$$

$\text{inv}\,\phi = .01869502$

$$t_{n1} + t_{n2} = \frac{(15 + 45)(.01869502 - .01736604) + \pi}{48}$$

$$= .0671$$

Sum of tooth thicknesses as originally specified = .0388 + .0327 = .0715

Reduction required for a minimum operating center distance of .6674 = .0715 − .0671 = .0044.

This reduction could be accomplished by reducing the normal circular tooth thickness of the 1st gear to .0368 and that of the 2nd gear to .0303.

Using Equation 9-6 it will be found that the reduced tooth thicknesses exceed the minimums required to avoid objectionable undercutting.

Given the gear data and operating center distance of a pair of mating helical gears; the recess action, the approach action and the contact ratio are determined by use of Equations 9-14, 9-15 and 9-16.

Equation 9-14

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2}$$

Equation 9-15

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2}$$

Equation 9-16

$$CR = \frac{(RA + AA) \times P_n}{\pi \times \cos\phi}$$

where:

RA = recess action
AA = approach action
CR = contact ratio
D_{o1} = outside diameter of driving gear
D_{o2} = outside diameter of driven gear
N_1 = number of teeth in driving gear
N_2 = number of teeth in driven gear
C = operating center distance
$Y = \frac{P_n \times \cos\psi}{\cos\phi}$
P_n = normal diametral pitch
ψ = helix angle
$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right)$

Example 9-9

The pair of helical gears in Example 9-8 have the data repeated below.

	1st Gear	2nd Gear
Number of teeth	15	45
Normal diametral pitch	48	48
Helix angle	18.600°	18.600°
Tooth form	PGT-1	PGT-1
Calc. normal cir. tooth thickness on std. pitch circle	.0388	.0327

It was determined that the gears would require to have a minimum operating center distance of .6731 for the normal circular tooth thicknesses specified above.

Find the outside diameters of both gears and then calculate the recess action, the approach action and the contact ratio.

(1) Find outside diameter of 1st gear.

$$D_{o1} = \frac{1}{P_n}\left(\frac{N_1}{\cos\psi} - 2.3158\right) + (2.7475 \times t_{n1})$$ Equ. 9-1

$P_n = 48 \qquad \cos\psi = \cos 18.600° = .94776841$

$N_1 = 15 \qquad t_{n1} = .0388$

$$D_{o1} = \frac{1}{48}\left(\frac{15}{.94776841} - 2.3158\right) + (2.7475 \times .0388)$$

$$= .3881$$

(2) Find outside diameter of 2nd gear.

$$D_{o2} = \frac{1}{P_n}\left(\frac{N_2}{\cos\psi} - 2.3158\right) + (2.7475 \times t_{n2})$$ Equ. 9-1

$N_2 = 45 \qquad t_{n2} = .0327$

$$D_{o2} = \frac{1}{48}\left(\frac{45}{.94776841} - 2.3158\right) + (2.7475 \times .0327)$$

$$= 1.0308$$

(3) Find the recess action

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2}$$ Equ. 9-14

$D_{o1} = .3881 \qquad N_1 = 15 \qquad N_2 = 45 \qquad C = .6731$

$$Y = \frac{P_n \times \cos\psi}{\cos\phi}$$

$P_n = 48 \qquad \psi = 18.600° \qquad \cos\psi = .94776841$

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right) = \tan^{-1}\left(\frac{.36397023}{.94776841}\right) = 21.00822°$$

$\cos\phi = .93352900$

$$Y = \frac{48 \times .94776841}{.93352900} = 48.7322$$

$$RA = \frac{.3881}{2}\sqrt{1 - \left(\frac{15}{.3881 \times 48.7322}\right)^2} - \frac{.6731 \times 15}{15 + 45}\sqrt{1 - \left(\frac{15 + 45}{2 \times .6731 \times 48.7322}\right)^2}$$

$$= .05014454$$

(4) Find the approach action

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 9-15}$$

$D_{o2} = 1.0308$

$$AA = \frac{1.0308}{2}\sqrt{1 - \left(\frac{45}{1.0308 \times 48.7322}\right)^2} - \frac{.6731 \times 45}{15 + 45}\sqrt{1 - \left(\frac{15 + 45}{2 \times .6731 \times 48.7322}\right)^2}$$

$$= .02490554$$

(5) Find the contact ratio

$$CR = \frac{(RA + AA) \times P_n}{\pi \times \cos\phi} \qquad \text{Equ. 9-16}$$

$RA = .05014442 \quad AA = .02490554 \quad P_n = 48$

$\pi = 3.1415926 \quad \cos\phi = .93352900$

$$CR = \frac{(.05014442 + .02490554) \times 48}{3.1415926 \times .93352900}$$

$$= 1.23$$

Given the normal circular tooth thickness of a helical gear, the measurement over two pins is determined by use of Equation 9-17.

Equation 9-17

(a) Gears having even number of teeth

$$M = \frac{N \times \cos\phi}{P_n \times \cos\psi \times \cos\phi_1} + d$$

(b) Gears having odd number of teeth

$$M = \left(\frac{N \times \cos\phi}{P_n \times \cos\psi \times \cos\phi_1}\right)\left(\cos\frac{90^\circ}{N}\right) + d$$

where:

M = measurement over two pins
N = number of teeth
P_n = normal diametral pitch
d = diameter of measuring pin
ψ = helix angle
$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right)$
ϕ_1 = angle whose involute is $\frac{P_n}{N}\left(t_n + \frac{d \times \cos\psi}{\cos\psi_1 \times \cos\phi}\right) + \text{inv}\,\phi - \frac{\pi}{N}$
t_n = normal circular tooth thickness
$\psi_1 = \tan^{-1}(\tan\psi \times \cos\phi)$
$\pi = 3.1415926$

Example 9-10

A helical gear has 45 teeth, a normal diametral pitch of 48, a helix angle of 18° 36′, the PGT-1 tooth form and a normal circular tooth thickness of .0327. Determine the measurement over two pins.

$$M = \left(\frac{N \times \cos\phi}{P_n \times \cos\psi \times \cos\phi_1}\right)\left(\cos\frac{90^\circ}{N}\right) + d \qquad \text{Equ. 9-17(b)}$$

$N = 45 \qquad P_n = 48 \qquad \psi = 18^\circ 36' \qquad \cos\psi = .94776841$

$$\phi = \tan^{-1}\left(\frac{.36397023}{.94776841}\right) = 21.008221 \quad \cos\phi = .93352900$$

$\text{inv}\,\phi = .01736604$

$$\text{inv}\,\phi_1 = \frac{P_n}{N}\left(t_n + \frac{d \times \cos\psi}{\cos\psi_1 \times \cos\phi}\right) + \text{inv}\,\phi - \frac{\pi}{N}$$

$t_n = .0327 \qquad \pi = 3.1415926 \qquad \text{Let } d = .036$

$$\psi_1 = \tan^{-1}(\tan\psi \times \cos\phi) = \tan^{-1}(.33653718 \times .93352900)$$

$$= 17.441010 \qquad \cos\psi_1 = .95402604$$

$$\text{inv}\,\phi_1 = \frac{48}{45}\left(.0327 + \frac{.036 \times .94776841}{.95402604 \times .93352900}\right) + .01736604 - \frac{3.1415926}{45}$$

$$= .02329730$$

$$\phi_1 = 23.078630 \qquad \cos\phi_1 = .91996777$$

$$M = \left(\frac{45 \times .93352900}{48 \times .94776841 \times .91996777}\right)(\cos 2^\circ) + .036$$

$$= (1.00374697 \times .99939082) + .036$$

$$= 1.0391$$

ADDITIONAL EXAMPLES

The PGT System is a tool for use in fashioning a gear design. The end result will depend upon how the individual engineer views the options open to him or her. Correctly used, the system will provide a design for gears having an adequate contact ratio, as high an efficiency as other considerations will permit, close to balanced tooth strength and sufficient clearance to insure that they will not bind when subjected to the extremes of the environment in which they will operate. Furthermore, the design will be presented in such a form that there can be no misinterpretation of the engineer's requirements, either by the molder of the gear or the inspector responsible for insuring that the requirements have been met.

There follow in this chapter two examples showing how one engineer used the system to arrive at the designs of gearing for two widely differing applications. In other hands the system could well produce designs that would differ, but be just as valid.

Example 10-1

A pair of gears is required for the first stage in a drive to a meter movement. The drive is speed-reducing and the ratio is 8:1. The center distance is fixed at .875. The pinion is driven through a small magnetic coupling, and the input torque is in the order of 2 gm. cms. It is essential, therefore, that the gearing be highly efficient.

The housing of the meter is to be molded of a glass-filled plastic having an internal lubricant. The spindles of the gears are to be molded integral with the gears and will run in holes molded in the housing, therefore there are no bearings involved. The operating center distance in the housing can be held to .878 max., .875 min.

The gears are to be molded of an acetal having a coefficient of linear thermal expansion of 4.7×10^{-5} in/in/°F. It is estimated that expansion of the acetal will not exceed .0002 in/in as the parts absorb moisture. The plastic of which the housing is to be molded has a coefficient of linear thermal expansion of 1.7×10^{-5} in/in/°F, and expansion due to moisture absorption is so small it can be ignored. The maximum temperature to which the instrument will be exposed is 150° F.

Design the pinion and gear.

(1) The first step is to settle upon the numbers of teeth in the pinion and gear and the diametral pitch. It is preferable that the pinion have at least 10 teeth. After reviewing the various possible combinations it would appear that a pinion having 14 teeth, a gear having 112 teeth and a diametral pitch of 72 might be a good first choice.

(2) Since the gearing is in the category of fine-pitch instrument drives, the PGT-4 tooth form is specified.

(3) Gears of the sizes involved, and molded of acetal in multi-cavity molds, can be held with ease to the accuracies required of AGMA Quality Class Q7. This quality is acceptable for gearing in commerical meters.

(4) Determine circular tooth thickness and outside diameters of pinion

$$t_1 = \frac{2.0787}{P} \qquad \text{Table 4-7}$$

where: t_1 = minimum tooth thickness of pinion

P = diametral pitch

$$t_1 = \frac{2.0787}{72}$$

$$= .0289$$

According to the AGMA "Gear Handbook" a tolerance of .0010 would be in order.

Circular tooth thickness of pinion = .0299 max., .0289 min.

$$D_{o1} = \frac{17.3296}{P}$$ Table 4-7

where: D_{o1} = outside diameter of pinion

$$= \frac{17.3296}{72}$$

$$= .2407$$

The outside diameter so obtained corresponds to the minimum tooth thickness of .0289. The maximum tooth thickness is greater than the minimum by .001, therefore the maximum outside diameter will exceed the minimum, theoretically, by $.0010 \times 2.7475$.

Outside diameter of pinion = .2434 max., .2407 min. (say .2440 max., .2410 min.)

(5) Determine the circular tooth thickness and the outside diameter of the gear. Since the operating center distance is fixed, the tooth thickness of the gear is found by use of Equation 5-3. To use Equation 5-3, a value must first be established for "C", the center distance at which pinion and gear will be in close mesh when their tooth thicknesses are at the allowable maximums. "C" is the minimum operating center distance less "Δ_C", as given by Equation 5-2.

$$\Delta_C = \frac{TCT_1 + TCT_2}{2} + C\left[(T-70)\left(\frac{COEF_1 \times N_1}{N_1+N_2} + \frac{COEF_2 \times N_2}{N_1+N_2} - COEF_H\right) + \left(\frac{M_1 \times N_1}{N_1+N_2} + \frac{M_2 \times N_2}{N_1+N_2} - M_H\right)\right] + \frac{TIR_1 + TIR_2}{2}$$

Equ. 5-2

To establish the total composite tolerances, TCT_1 and TCT_2, the pitch diameters must be known.

$$D = \frac{N}{P}$$ Equ. 2-1

$$D_1 = \frac{14}{72} = .1944$$

$$D_2 = \frac{112}{72} = 1.5556$$

Standard pitch diameter of pinion = .1944
Standard pitch diameter of gear = 1.5556

According to the AGMA "Gear Handbook" the pinion and gear have maximum total composite tolerances of .0022 and .0026 respectively.

$TCT_1 = .0022$ $TCT_2 = .0026$
$C = .875$ $T = 150$ $COEF_1 = 4.7 \times 10^{-5}$ $COEF_2 = 4.7 \times 10^{-5}$
$COEF_H = 1.7 \times 10^{-5}$ $M_1 = .0002$ $M_2 = .0002$ $M_H = 0$ $TIR_1 = 0$ $TIR_2 = 0$
$N_1 = 14$ $N_2 = 112$

$$\Delta_C = \frac{.0022 + .0026}{2} + .875\left[(150-70)\left(\frac{.000047 \times 14}{14+112} + \frac{.000047 \times 112}{14+112} - .000017\right) + \left(\frac{.0002 \times 14}{14+112} + \frac{.0002 \times 112}{14+112} - 0\right)\right] + \frac{0+0}{2}$$

Equ. 5-2

$$= .0047$$

Value for "C" in Equation 5-3 = .875 − .0047 = .8703

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 6-2}$$

$D_{o2} = 1.5605$

$$AA = \frac{1.5605}{2}\sqrt{1 - \left(\frac{112}{1.5605 \times 76.6208}\right)^2} - \frac{.8765 \times 112}{14 + 112}\sqrt{1 - \left(\frac{14 + 112}{2 \times .8765 \times 76.6208}\right)^2}$$

$= .00327031$

$CR = (RA + AA) \div (2.9521314 \div P)$ Equ. 6-3

$RA = .04598270 \qquad AA = .00327031 \qquad P = 72$

$CR = (.04598270 + .00327031) \div (2.9521314 \div 72)$

$= 1.201$

$$\text{Percentage recess action} = \frac{.04598270 \times 100}{.04598270 + .00327031}$$

$= 93\%$

The contact ratio is adequate and the percentage of recess action is excellent, but it is possible that an improvement could be effected. Try a pinion having 15 teeth, a gear having 120 teeth and a diametral pitch of 76.

(7) Determine circular tooth thickness and outside diameter of pinion.

$$t_1 = \frac{2.0361}{P} \qquad \text{Table 4-7}$$

$P = 76$

$$t_1 = \frac{2.0361}{76}$$

$= .0268$

Circular tooth thickness of pinion = .0278 max., .0268 min.

$$D_{o1} = \frac{18.2996}{P} \qquad \text{Table 4-7}$$

$$= \frac{18.2996}{76}$$

$= .2408$

Outside diameter of pinion = .2440 max., .2410 min.

$$t_1 + t_2 = \frac{(N_1 + N_2)(\text{inv}\,\phi_1 - .01490438) + \pi}{P}$$ Equ. 5-3

$$\phi_1 = \cos^{-1}\left[\frac{(N_1 + N_2) \times .46984631}{P \times C}\right]$$

$N_1 = 14 \quad N_2 = 112 \quad P = 72 \quad C = .8703 \quad \pi = 3.1415926$

$$\phi_1 = \cos^{-1}\left[\frac{(14 + 112) \times .46984631}{72 \times .8703}\right]$$

$$= \cos^{-1}\left[.94476737\right]$$

$$= 19.131762^\circ$$

$$\text{inv}\,\phi_1 = .01298975$$

$$t_1 + t_2 = \frac{(14 + 112)(.01298975 - .01490438) + 3.1415926}{72}$$

$$= .0403$$

$$t_2 = .0403 - .0299 = .0104$$

Circular tooth thickness of gear = .0104 max., .0094 min.

$$D_o = \frac{1}{P}(N - 1.6158) + (2.7475 \times t)$$ Equ. 4-4

(i) $D_{o2} = \frac{1}{72}(112 - 1.6158) + (2.7475 \times .0104) = 1.5617$

(ii) $D_{o2} = \frac{1}{72}(112 - 1.6158) + (2.7475 \times .0094) = 1.5589$

It is not essential to adhere strictly to the maximum and minimum outside diameters as given by Equation 4-4. They may be adjusted at the discretion of the designer to avoid unnecessarily close tolerances. In this example let the maximum outside diameter be 1.5630 and the minimum 1.5580.

(6) At this stage in the design it is advisable to calculate the contact ratio and percentage of recess action. Mean values of the outside diameters and operating center distance may be used in Equations 6-1, 6-2 and 6-3.

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2}$$ Equ. 6-1

$D_{o1} = .2425 \quad N_2 = 112 \quad C = .8765 \quad N_1 = 14$

$$Y = \frac{P}{.93969262} = \frac{72}{.93969262} = 76.6208$$

$$RA = \frac{.2425}{2}\sqrt{1 - \left(\frac{14}{.2425 \times 76.6208}\right)^2} - \frac{.8765 \times 14}{14 + 112}\sqrt{1 - \left(\frac{14 + 112}{2 \times .8765 \times 76.6208}\right)^2}$$

$$= .04598270$$

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 6-2}$$

$D_{o2} = 1.5605$

$$AA = \frac{1.5605}{2}\sqrt{1 - \left(\frac{112}{1.5605 \times 76.6208}\right)^2} - \frac{.8765 \times 112}{14 + 112}\sqrt{1 - \left(\frac{14 + 112}{2 \times .8765 \times 76.6208}\right)^2}$$

$= .00327031$

$CR = (RA + AA) \div (2.9521314 \div P)$ Equ. 6-3

$RA = .04598270 \qquad AA = .00327031 \qquad P = 72$

$CR = (.04598270 + .00327031) \div (2.9521314 \div 72)$

$= 1.201$

$$\text{Percentage recess action} = \frac{.04598270 \times 100}{.04598270 + .00327031}$$

$= 93\%$

The contact ratio is adequate and the percentage of recess action is excellent, but it is possible that an improvement could be effected. Try a pinion having 15 teeth, a gear having 120 teeth and a diametral pitch of 76.

(7) Determine circular tooth thickness and outside diameter of pinion.

$$t_1 = \frac{2.0361}{P} \qquad \text{Table 4-7}$$

$P = 76$

$$t_1 = \frac{2.0361}{76}$$

$= .0268$

Circular tooth thickness of pinion = .0278 max., .0268 min.

$$D_{o1} = \frac{18.2996}{P} \qquad \text{Table 4-7}$$

$$= \frac{18.2996}{76}$$

$= .2408$

Outside diameter of pinion = .2440 max., .2410 min.

(8) Determine circular tooth thickness and outside diameter of gear.

$$D_1 = \frac{N_1}{P}$$

$N_1 = 15 \quad P = 76$

$$D_1 = \frac{15}{76} = .1974$$

$$D_2 = \frac{N_2}{P}$$

$N_2 = 120$

$$D_2 = \frac{120}{76} = 1.5789$$

Standard pitch diameter of pinion = .1974
Standard pitch diameter of gear = 1.5789

$TCT_1 = .0021 \quad TCT_2 = .0025$

$$\Delta_C = \frac{.0021 + .0025}{2} + .875\left[(150 - 70)\left(\frac{.000047 \times 15}{15 + 120} + \frac{.000047 \times 120}{15 + 120} - .000017\right) + \left(\frac{.0002 \times 15}{15 + 120} + \frac{.0002 \times 112}{15 + 120} - 0\right)\right] + \frac{0 + 0}{2}$$

$= .0046$ Equ. 5-2

Value for "C" in Equation 5-3 = .875 − .0046 = .8704

$$t_1 + t_2 = \frac{(N_1 + N_2)(\text{inv}\,\phi_1 - .01490438) + \pi}{P} \qquad \text{Equ. 5-3}$$

$N_1 = 15 \quad N_2 = 120 \quad P = 76 \quad C = .8704 \quad \pi = 3.1415926$

$$\phi_1 = \cos^{-1}\left[\frac{(15 + 120) \times .46984631}{76 \times .8704}\right]$$

$$= \cos^{-1}\left[.95886422\right]$$

$$= 16.491023°$$

$\text{inv}\,\phi_1 = .00822043$

$$t_1 + t_2 = \frac{(15 + 120)(.00822043 - .01490438) + 3.1415926}{76}$$

$$= .0295$$

$t_2 = .0295 - .0278 = .0017$

Circular tooth thickness of gear = .0017 max., .0007 min.

(i) $D_{o2} = \frac{1}{76}(120 - 1.6158) + (2.7475 \times .0017) = 1.5624$

(ii) $D_{o2} = \frac{1}{76}(120 - 1.6158) + (2.7475 \times .0007) = 1.5596$

Outside diameter of gear = 1.5630 max. 1.5580 min.

(9) Calculate the contact ratio and percentage of recess action.

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 6-1}$$

$D_{o1} = .2425 \quad N_1 = 15 \quad N_2 = 120 \quad C = .8765$

$$Y = \frac{P}{.93969262} = \frac{76}{.93969262} = 80.8775$$

$$RA = \frac{.2425}{2}\sqrt{1 - \left(\frac{15}{.2425 \times 80.8775}\right)^2} - \frac{.8765 \times 15}{15 + 120}\sqrt{1 - \left(\frac{15 + 120}{2 \times .8765 \times 80.8775}\right)^2}$$

$$= .04836388$$

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 6-2}$$

$D_{o2} = 1.5605$

$$AA = \frac{1.5605}{2}\sqrt{1 - \left(\frac{120}{1.5605 \times 80.8775}\right)^2} - \frac{.8765 \times 120}{15 + 120}\sqrt{1 - \left(\frac{15 + 120}{2 \times .8765 \times 80.8775}\right)^2}$$

$$= .00370185$$

$$CR = (RA + AA) \times .3387383 \times P$$

$RA = .04836388 \quad AA = .00370185 \quad P = 76$

$$CR = (.04836388 + .00370185) \times .3387383 \times 76 = 1.340$$

$$\text{Percentage recess action} = \frac{.04836388 \times 100}{.04836388 + .00370185} = 93\%$$

(10) The second pair is an improvement over the first choice: the percentage of recess action is unchanged at 93, but the contact ratio is increased from 1.20 to 1.34, providing a smoother and more even drive, and, what is important when maximum efficiency is being sought, the operating pressure angle is reduced to 16.49° from 19.13°. The relationship between gear efficiency and operating pressure angle is discussed in Chapt. 1.

(11) All that now remains is to establish the gear data that are to appear on the drawings of the pinion and gear. The following are the data already determined:

	Pinion	Gear
Number of teeth	15	120
Diametral pitch	76	76
Pressure angle	20°	20°
Std. pitch diameter	.1974	1.5789
Tooth form	PGT-4	PGT-4
Calc. cir. tooth thickness on std. pitch circle	.0278 – .0268	.0017 – .0007
AGMA Quality Number	Q7	Q7
Max. total composite tolerance	.0021	.0025
Outside diameter	.2440 – .2410	1.5630 – 1.5580
Operating center distance	.878 – .875	

A reference to Fig. 7-1 will show what additional data are required.

(12) Addendum.

The addendum is that of the PGT-4 tooth form.

$$\text{Addendum} = \frac{1.35}{P} = \frac{1.35}{76} = .0178$$ Fig. 3-4

(13) Whole depth.

The whole depth is also that of the PGT-4 tooth form.

$$\text{Whole depth} = \frac{3.03}{P} = \frac{3.03}{76} = .0399$$ Fig. 3-4

(14) Gear testing radii.

(i) Assume the master gear will have 152 teeth and the standard circular tooth thickness of .0207.

$$\text{Std. pitch diameter of master gear} = \frac{N}{P} = \frac{152}{76} = 2.0000$$

(ii) Find the close mesh center distance of the pinion and master for the maximum pinion tooth thickness of .0278.

$$C = \frac{(N_1 + N_2) \times .46984631}{P \times \cos\phi_1}$$ Equ. 5-1

$N_1 = 15 \quad N_2 = 152 \quad P = 76$

$$\operatorname{inv}\phi_1 = \frac{P(t_1 + t_2) - \pi}{N_1 + N_2} + .01490438$$

$t_1 = .0278 \quad t_2 = .0207 \quad \pi = 3.1415926$

$$\operatorname{inv}\phi_1 = \frac{76(.0278 + .0207) - 3.1415926}{15 + 152} + .01490438 = .01816430$$

$$\phi_1 = 21.313455°$$

$$\cos\phi_1 = .93160590$$

$$C = \frac{(15 + 152) \times .46984631}{76 \times .93160590} = 1.082$$

(iii) Repeat for minimum pinion tooth thickness of .0268.

$$\text{inv}\,\phi_1 = \frac{76\,(.0268 + .0207) - 3.1415926}{15 + 152} + .01490438 \qquad \text{Equ. 5-1}$$

$$= .01770921$$

$$\phi_1 = 21.140624$$

$$\cos\phi_1 = .93269805$$

$$C = \frac{(15 + 152) \times .46984631}{76 \times .93269805}$$

$$= 1.069$$

(iv) Find the close mesh center distance of gear and master for the maximum gear tooth thickness of .0017.

$$C = \frac{(N_1 + N_2) \times .46984631}{P \times \cos\phi_1} \qquad \text{Equ. 5-1}$$

$$N_1 = 120 \qquad N_2 = 152 \qquad P = 76$$

$$\text{inv}\,\phi_1 = \frac{P\,(t_1 + t_2) - \pi}{N_1 + N_2} + .01490438$$

$$t_1 = .0017 \qquad t_2 = .0207$$

$$\text{inv}\,\phi_1 = \frac{76\,(.0017 + .0207) - 3.1415926}{120 + 152} + .01490438$$

$$= .00961323$$

$$\phi_1 = 17.352909^\circ$$

$$\cos\phi_1 = .95448579$$

$$C = \frac{(120 + 152) \times .46984631}{76 \times .95448579}$$

$$= 1.7617$$

(v) Repeat for minimum gear tooth thickness of .0007

$$\text{inv}\,\phi_1 = \frac{76\,(.0007 + .0207) - 3.1415926}{120 + 152} + .01490438$$

$$= .00933382$$

$$\phi_1 = 17.187280$$

$$\cos\phi_1 = .95534399$$

$$C = \frac{(120 + 152) \times .46984631}{76 \times .95534399}$$

$$= 1.7602$$

(vi) Pitch diameter of master gear $= 2.0000$

Total composite tolerance of pinion $= .0021$

Max. testing radius of pinion $= 1.1082 + \frac{.0021}{2} - \frac{2.0000}{2}$

$= .1093$

Min. testing radius of pinion $= 1.1069 - \frac{.0021}{2} - \frac{2.0000}{2}$

$= .1059$

Total composite tolerance of gear $= .0025$

Max. testing radius of gear $= 1.7617 + \frac{.0025}{2} - \frac{2.0000}{2}$

$= .7630$

Min. testing radius of gear $= 1.7602 - \frac{.0025}{2} - \frac{2.0000}{2}$

$= .7590$

(15) Maximum tooth-to-tooth composite tolerance.

From the AGMA "Gear Handbook" it is found that maximum tooth-to-tooth tolerances of the pinion and gear are .0015 and .0012 respectively.

(16) Testing pressure.

According to the AGMA "Gear Handbook" the testing pressure recommended for non-metallic gears having diametral pitches of 60-79 should range from 4.5 to 7.5 ounces. Let the testing pressure be 5 ounces.

(17) Measurement over pins.

(i) Find measurement over pins of pinion for maximum tooth thickness of .0278.

$$M = \frac{.93969262 \times N}{P \times \cos\phi_1}\left(\cos\frac{90^\circ}{N}\right) + d \qquad \text{Equ. 7-1(b)}$$

$N = 15 \quad P = 76 \quad \text{Let } d = .025$

$$\text{inv}\,\phi_1 = \frac{P}{N}\left[t + (1.0641778 \times d)\right] + .01490438 - \frac{\pi}{N}$$

$t = .0278 \quad \pi = 3.1415926$

$$\text{inv}\,\phi_1 = \frac{76}{15}\left[.0278 + (1.0641778 \times .025)\right] + .01490438 - \frac{3.1415926}{15}$$

$= .08111406$

$\phi_1 = 34.002202$

$\cos\phi_1 = .82901608$

$$M = \frac{.93969262 \times 15}{76 \times .82901608}\left(\cos\frac{90^\circ}{15}\right) + .025$$

$= (.22371779 \times .99452189) + .025$

$= .2475$

(ii) Repeat for minimum tooth thickness of .0268.

$$\operatorname{inv}\phi_1 = \frac{76}{15}\left[.0268 + (1.0641778 \times .025)\right] + .01490438 - .20943951$$

$$= .07604739$$

$$\phi_1 = 33.348352$$

$$\cos\phi^1 = .83534374$$

$$M = \frac{.93969262 \times 15}{76 \times .83534374}\left(\cos\frac{90°}{15}\right) + .025$$

$$= .2458$$

(iii) Measurement over two pins of pinion = .2475 max., .2458 min.

(iv) Find measurement over two pins of gear for maximum tooth thickness of .0017.

$$M = \frac{.93969262 \times N}{P \times \cos\phi_1} + d \qquad \text{Equ. 7-1(a)}$$

$$N = 120 \qquad P = 76 \qquad d = .025$$

$$\operatorname{inv}\phi_1 = \frac{76}{120}\left[.0017 + (1.0641778 \times .025)\right] + .01490438 - \frac{3.1415926}{120}$$

$$= .00665059$$

$$\phi_1 = 15.388970$$

$$\cos\phi_1 = .96414651$$

$$M = \frac{.93969262 \times 120}{76 \times .96414651} + .025$$

$$= 1.5639$$

The maximum outside diameter of the gear is 1.5630. To insure that an accurate measurement over pins is obtainable, it is preferable to increase diameter of pin from .025 to, say, .028.

$$\operatorname{inv}\phi_1 = \frac{76}{120}\left[.0017 + (1.0641778 \times .028)\right] + .01490438 - \frac{3.1415926}{120}$$

$$= .00867253$$

$$\phi_1 = 16.781146$$

$$\cos\phi_1 = .95741456$$

$$M = \frac{.93969262 \times 120}{76 \times .95741456} + .028$$

$$= 1.5777$$

(v) Repeat for minimum tooth thickness of .0007.

$$\text{inv}\,\phi_1 = \frac{76}{120}\left[.0007 + (1.0641778 \times .028)\right] + .01490438 - \frac{3.1415926}{120}$$

$$= .00803919$$

$$\phi_1 = 16.371626$$

$$\cos\phi_1 = .95945368$$

$$M = \frac{.93969262 \times 120}{76 \times .95945368} + .028$$

$$= 1.5744$$

(vi) Measurement over two pins of gear = 1.5777 max.. 1.5744 min.

(18) Maximum root diameters of pinion and gear.

$$D_R = \frac{1}{P}(N - 7.6758) + (2.7475 \times t)$$ Equ. 4-8

(i) Pinion

$P = 76 \quad N = 15 \quad t = .0278$

$$D_R = \frac{1}{76}(15 - 7.6758) + (2.7475 \times .0278)$$

$$= .1728$$

(ii) Gear

$P = 76 \quad N = 120 \quad t = .0017$

$$D_R = \frac{1}{76}(120 - 7.6758) + (2.7475 \times .0017)$$

$$= 1.4826$$

The data to go on the drawings are now complete and are listed on the drawings as shown in Figs. 10-1 and 10-2.

SPUR & HELICAL GEARS

FIG. 10-1

SPUR GEAR DATA		
BASIC SPECIFICATIONS	NUMBER OF TEETH	15
	DIAMETRAL PITCH	76
	PRESSURE ANGLE	20°
	STANDARD PITCH DIAMETER	.1974
	TOOTH FORM	PGT-4
	ADDENDUM	.0178
	WHOLE DEPTH	.0399
	CALC. CIR. TOOTH THICKNESS ON STD. PITCH CIRCLE	.0278 MAX. .0268 MIN.
MANUFACTURING AND INSPECTION	GEAR TESTING RADIUS	.1093 MAX. .1059 MIN.
	AGMA QUALITY NUMBER	Q7
	MAX. TOTAL COMPOSITE TOLERANCE	.0021
	MAX. TOOTH-TO-TOOTH COMPOSITE TOLERANCE	.0015
	MASTER GEAR SPECIFICATIONS	152T, .0207CTT
	TESTING PRESSURE (OUNCES)	5
	DIAMETER OF MEASURING PIN	.025
	MEASUREMENT OVER TWO PINS (FOR SETUP ONLY)	.2475 MAX. .2458 MIN.
	OUTSIDE DIAMETER	$.244 {+.000 \atop -.003}$
	MAX. ROOT DIAMETER	.1728
ENGINEERING REFERENCES	MATING GEAR PART NUMBER	GEAR
	NUMBER OF TEETH IN MATING GEAR	120
	OPERATING CENTER DISTANCE	.8780 MAX. .8750 MIN.

FIG. 10-2

SPUR GEAR DATA		
BASIC SPECIFICATIONS	NUMBER OF TEETH	120
	DIAMETRAL PITCH	76
	PRESSURE ANGLE	20°
	STANDARD PITCH DIAMETER	1.5789
	TOOTH FORM	PGT-4
	ADDENDUM	.0178
	WHOLE DEPTH	.0399
	CALC. CIR. TOOTH THICKNESS ON STD. PITCH CIRCLE	.0017 MAX. .0007 MIN.
MANUFACTURING AND INSPECTION	GEAR TESTING RADIUS	.7630 MAX. .7590 MIN.
	AGMA QUALITY NUMBER	Q7
	MAX. TOTAL COMPOSITE TOLERANCE	.0025
	MAX. TOOTH-TO-TOOTH COMPOSITE TOLERANCE	.0012
	MASTER GEAR SPECIFICATIONS	152T, .0207CTT
	TESTING PRESSURE (OUNCES)	5
	DIAMETER OF MEASURING PIN	.028
	MEASUREMENT OVER TWO PINS (FOR SETUP ONLY)	1.5777 MAX. 1.5744 MIN.
	OUTSIDE DIAMETER	$1.563 {}^{+.000}_{-.005}$
	MAX. ROOT DIAMETER	1.4826
ENGINEERING REFERENCES	MATING GEAR PART NUMBER	PINION
	NUMBER OF TEETH IN MATING GEAR	15
	OPERATING CENTER DISTANCE	.8780 MAX. .8750 MIN.

Example 10-2

A pinion and gear are required for a speed-reducing power drive in a domestic appliance. The ratio is 3:1. Because quiet operation is of importance, the gears are to be helical. A preliminary estimate indicates that a pinion and gear having 15 and 45 teeth respectively, a normal diametral pitch of 16 and a helix angle of 18°, would be capable of transmitting the load involved.

The environment in which the drive will operate is such that the gears could be subjected to a maximum temperature of 175° F and to a relative humidity of 80 percent.

The housing in which the gears are mounted is of aluminum, and the bearings carrying the gear shafts have a maximum allowable runout of .0005 *T.I.R.*

The plastic of which the pinion is to be molded has a coefficient of linear thermal expansion of 4.5×10^{-5} in/in/° F, and expansion due to moisture pick-up is estimated to be .001 in/in. The values of the corresponding properties of the plastic of which the gear is to be molded are 2.5×10^{-5} in/in/° F and .0005 in/in. The aluminum of the housing has a coefficient of linear thermal expansion of 1.0×10^{-5} in/in/° F.

Design the pinion and gear and determine the data to go on the drawings.

(1) Find normal circular tooth thicknesses of pinion and gear. For helical gears in a power drive the PGT-1 tooth form is specified, and the teeth are designed to have balanced strength. The equations for balanced tooth strength are 9-8, 9-9 and 9-10.

Find a value for N

$$N = \frac{2.0938 \times \cos\psi}{1 - \cos\phi}$$

$$\psi = 18° \qquad \cos\psi = .95105652$$

$$\phi = \tan^{-1}\left(\frac{.36397023}{\cos\psi}\right) = \tan^{-1}\left(\frac{.36397023}{.95105652}\right) = 20.941896°$$

$$\cos\phi = .93394337$$

$$N = \frac{2.0938 \times .95105652}{1 - .93394337}$$

$$= 30.146$$

The pinion has less than 30.146 teeth and the gear has more, therefore 9-9 is the equation to use

$$t_{n1} = \frac{1}{P_n}\left[2.3329 - \frac{.36397023 \times N_1 \times (1 - \cos\phi)}{\cos\psi}\right] \qquad \text{Equ. 9-9(a)}$$

$$P_n = 16 \qquad N_1 = 15 \qquad \cos\phi = .93394337 \qquad \cos\psi = .95105652$$

$$t_{n1} = \frac{1}{16}\left[2.3329 - \frac{.36397023 \times 15 \times (1 - .93394337)}{.95105652}\right]$$

$$= .1221$$

Normal circular tooth thickness of pinion = .1221

$$t_{n2} = \frac{N_2}{P_n} \left[\frac{P_n \times B \times \cos\psi}{N_2 - (2.0938 \times \cos\psi)} + \text{inv}\,\phi_2 - \text{inv}\,\phi \right] \qquad \text{Equ. 9-9(b)}$$

$N_2 = 45 \quad P_n = 16 \quad \cos\psi = .95105652$

$\phi = 20.941896° \quad \text{inv}\,\phi = .01719592$

$$\phi_2 = \cos^{-1}\left[\frac{N_2 \times \cos\phi}{N_2 - (2.0938 \times \cos\psi)}\right]$$

$$= \cos^{-1}\left[\frac{45 \times .93394337}{45 - (2.0938 \times .95105652)}\right]$$

$$= \cos^{-1}\,(.97718539)$$

$$= 12.262338°$$

$\text{inv}\,\phi_2 = .00332862$

$$B = \frac{N_1 \times \cos\phi}{P_n \times \cos\psi}\left(\frac{P_n \times t_{n1}}{N_1} + \text{inv}\,\phi\right)$$

$t_{n1} = .1221 \quad N_1 = 15$

$$B = \frac{15 \times .93394337}{16 \times .95105652}\left(\frac{16 \times .1221}{15} + .01719592\right)$$

$$= .13573405$$

$$t_{n2} = \frac{45}{16}\left[\frac{16 \times .13573405 \times .95105652}{45 - (2.0938 \times .95105652)} + .00332862 - .01719592\right]$$

$$= .0961$$

Normal circular tooth thickness of gear = .0961

The tooth thicknesses thus calculated are maximum values.

The tolerances to be applied will be determined later.

(2) Find the close mesh center distance by using Equation 9-11

$$C = \frac{(N_1 + N_2) \times \cos\phi}{2 \times P_n \times \cos\psi \times \cos\phi_1} \qquad \text{Equ. 9-11}$$

$$\text{inv}\,\phi_1 = \frac{P_n(t_{n1} + t_{n2}) - \pi}{N_1 + N_2} + \text{inv}\,\phi$$

$t_{n1} = .1221 \quad t_{n2} = .0961 \quad \pi = 3.1415926$

$$\text{inv}\,\phi_1 = \frac{16\,(.1221 + .0961) - 3.1415926}{15 + 45} + .01719592$$

$$= .02302271$$

$\phi_1 = 22.991607°$

$\cos\phi_1 = .92056208$

$$C = \frac{(15 + 45) \times .93394337}{2 \times 16 \times .95105652 \times .92056208} = 2.0001$$

(3) Find Δ_C by use of Equation 9-12

$$\Delta_C = \frac{TCT_1 + TCT_2}{2} + C\left[(T-70)\left(\frac{COEF_1 \times N_1}{N_1 + N_2} + \frac{COEF_2 \times N_2}{N_1 + N_2} - COEF_H\right) + \left(\frac{M_1 \times N_1}{N_1 + N_2} + \frac{M_2 \times N_2}{N_1 + N_2} - M_H\right)\right] + \frac{TIR_1 + TIR_2}{2}$$

Equ. 9-12

The AGMA Quality Numbers recommended in the AGMA "Gear Handbook" range from Q6 to Q8 inclusive. Let the Quality Number for both pinion and gear be Q7. To establish the maximum total composite tolerances to be specified for the pinion and gear it is necessary to know their standard pitch diameters.

$$D = \frac{N}{P_n \times \cos\psi}$$

$$D_1 = \frac{15}{16 \times .95105652}$$

$$= .9857$$

$$D_2 = \frac{45}{16 \times .95105652}$$

$$= 2.9572$$

Standard pitch diameter of pinion = .9857
Standard pitch diameter of gear = 2.9572

According to the AGMA "Gear Manual" the pinion and gear have maximum total composite tolerances of .0043 and .0047 respectively.

$TCT_1 = .0043 \quad TCT_2 = .0047$

$C = 2.0001 \quad T = 175 \quad COEF_1 = 4.5 \times 10^{-5} \quad COEF_2 = 2.5 \times 10^{-5}$

$COEF_H = 1.0 \times 10^{-5} \quad M_1 = .0010 \quad M_2 = .0005 \quad M_H = 0 \quad TIR_1 = .0005 \quad TIR_2 = .0005$

$$\Delta_C = \frac{.0043 + .0047}{2} + 2.0001\left[(175-70)\left(\frac{.000045 \times 15}{15 + 45} + \frac{.000025 \times 45}{15 + 45} - .00001\right) + \left(\frac{.0010 \times 15}{15 + 45} + \frac{.0005 \times 45}{15 + 45} - 0\right)\right] + \frac{.0005 + .0005}{2}$$

$$= .0105$$

(4) Determine the operating center distance

Minimum operating center distance $= C + \Delta_C$

$$= 2.0001 + .0105$$

$$= 2.0106$$

The center distance tolerance should be arrived at in consultation with the manufacturing division responsible for the design of the housing. Assume the operating center distance is established at 2.0160 max., 2.0110 min.

(5) At this stage in the design it is advisable to calculate the recess action, approach action and contact ratio by using Equations 9-14, 9-15 and 9-16. To use the equations it is necessary to establish maximum and minimum values for the outside diameters of the pinion and gear.

The maximum normal circular tooth thicknesses of the pinion and gear are .1221 and .0961 respectively. According to the AGMA Gear **Handbook**, it is recommended that the tooth thickness tolerance of Quality Q7 gears having a diametral pitch of 16 range from .0027 to .0004. Let the tolerance be .0020.

Normal circular tooth thickness of pinion = .1221 max., .1201 min.
Normal circular tooth thickness of gear = .0961 max., .0941 min.

The outside diameter of a molded gear is related directly to the tooth thickness, and is calculated by using Equ. 9-1 in the case of a helical gear.

$$D_o = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 2.3158\right) + (2.7475 \times t_n) \qquad \text{Equ. 9-1}$$

(a) Pinion

$P_n = 16 \quad N = 15 \quad \cos\psi = .95105652$

$t_n = .1221$ max., .1201 min.

$$D_o = \frac{1}{16}\left(\frac{15}{.95105652} - 2.3158\right) + (2.7475 \times .1221)$$
$$= 1.1765 \text{ max.}$$

$$D_o = \frac{1}{16}\left(\frac{15}{.95105652} - 2.3158\right) + (2.7475 \times .1201)$$
$$= 1.1710 \text{ min.}$$

(b) Gear

$N = 45 \quad t_n = .0961$ max., .0941 min.

$$D_o = \frac{1}{16}\left(\frac{45}{.95105652} - 2.3158\right) + (2.7475 \times .0961)$$
$$= 3.0765 \text{ max.}$$

$$D_o = \frac{1}{16}\left(\frac{45}{.95105652} - 2.3158\right) + (2.7475 \times .0941)$$
$$= 3.0710 \text{ min.}$$

Outside diameter of pinion = 1.1765 max., 1.1710 min.
Outside diameter of gear = 3.0765 max., 3.0710 min.

In using Equations 9-14 and 9-15 it is usually sufficient to use the mean values of the outside diameters of the gears and the mean value of the operating center distance, because variations in dimensions within the allowed tolerances will tend to neutralise themselves; but if unusually generous tolerances have been applied, it is advisable to check their possible effect by calculating the minimum contact ratio. The minimum contact ratio will obtain when the outside diameters of the gears are at the minimums allowed by the tolerances and the operating center distance at the maximum allowed by its tolerance.

$$RA = \frac{D_{o1}}{2}\sqrt{1 - \left(\frac{N_1}{D_{o1} \times Y}\right)^2} - \frac{C \times N_1}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 9-14}$$

D_{o1} = 1.1765 max., 1.1710 min. = 1.17375 mean

D_{o2} = 3.0765 max., 3.0710 min. = 3.07375 mean

C = 2.0160 max., 2.0110 min. = 2.0135 mean

$$Y = \frac{P_n \times \cos\psi}{\cos\phi}$$

$$= \frac{16 \times .95105652}{.93394337}$$

$$= 16.2932$$

$$RA = \frac{1.17375}{2}\sqrt{1 - \left(\frac{15}{1.17375 \times 16.2932}\right)^2} - \frac{2.0135 \times 15}{15 + 45}\sqrt{1 - \left(\frac{15 + 45}{2 \times 2.0135 \times 16.2932}\right)^2}$$

$$= .16034281$$

$$AA = \frac{D_{o2}}{2}\sqrt{1 - \left(\frac{N_2}{D_{o2} \times Y}\right)^2} - \frac{C \times N_2}{N_1 + N_2}\sqrt{1 - \left(\frac{N_1 + N_2}{2 \times C \times Y}\right)^2} \qquad \text{Equ. 9-15}$$

$$= \frac{3.07375}{2}\sqrt{1 - \left(\frac{45}{3.07375 \times 16.2932}\right)^2} - \frac{2.0135 \times 45}{15 + 45}\sqrt{1 - \left(\frac{15 + 45}{2 \times 2.0135 \times 16.2932}\right)^2}$$

$$= .06339804$$

$$CR = \frac{(RA + AA) \times P_n}{\pi \times \cos\phi}$$

$$= \frac{(.16034281 + .06339804) \times 16}{3.1415926 \times .93394337}$$

$$= 1.220$$

$$\text{Percentage recess action} = \frac{.16034281 \times 100}{.16034281 + .06339804}$$

$$= 72\%$$

HELICAL OVERLAP

A pair of helical gears has an additional contact ratio, referred to as the helical overlap. The amount of helical overlap is dependant upon the face width of the gears. The face width of a helical gear should be, ideally, twice the axial pitch, if full benefit is to be obtained from the helical action, but this is not always possible. The face width should not exceed the pitch diameter, otherwise torsional twist will cause the load to concentrate at one end.

The face width of the pinion, in this example, should not exceed .9857, its pitch diameter. Let the face width be .800 ± .005.

The axial pitch of a helical gear is the lead divided by the number of teeth. Find the lead and then the axial pitch of the pinion.

$$L = \pi \times D \times \cot \psi \qquad \text{Equ. 8-1}$$

$$\pi = 3.1415926 \quad D = .9857 \quad \psi = 18^\circ \quad \cot \psi = 3.0776836$$

$$L = 3.1415926 \times .9857 \times 3.0776836$$

$$= 9.5310$$

$$\text{Axial pitch} = \frac{L}{N} \quad L = 9.5310 \quad N = 15$$

$$\text{Axial pitch} = \frac{9.5310}{15} = .6354$$

$$\text{Helical overlap} = \frac{\text{Face}}{\text{Axial pitch}}$$

$$\text{Helical overlap} = \frac{.800}{.6354}$$

$$= 1.26$$

The contact ratio is 1.22 and the helical overlap is 1.26, which together provide a total contact ratio of 2.48. A total contact ratio of 2.48 coupled with a recess action of 72 percent will result in a smooth, quiet drive. No adjustments to the design of the gears are necessary.

(6) The data so far established can now be listed.

	Pinion	Gear
Number of teeth	15	45
Normal diametral pitch	16	16
Normal pressure angle	20°	20°
Helix angle	18°	18°
Standard pitch diameter	.9857	2.9572
Tooth form	PGT-1	PGT-1
Calc. normal cir. tooth thickness on std. pitch circle	.1221 max. .1201 min.	.0961 max. .0941 min.
AGMA Quality Number	Q7	Q7
Max. total composite tolerance	.0043	.0047
Lead	9.5310	
Outside diameter	$1.1765^{+.0000}_{-.0055}$	$3.0765^{+.0000}_{-.0055}$
Operating center distance	2.0160 max. 2.0110 min.	

A reference to Fig. 8-3 will show what drawing data remains to be determined.

(7) Establish hand of helix.

This is a designer's choice and will depend upon how the end thrust is to be taken up. Assume that the pinion in this example has a right hand helix. The gear must then have a left hand helix.

(8) Establish addendum.

The addendum to be specified is that of the PGT-1 tooth form.

$$\text{Addendum} = \frac{1}{P} = \frac{1}{16} = .0625$$ Fig. 3-1

(9) Establish whole depth.

The whole depth to be specified is also that of the PGT-1 tooth form.

$$\text{Whole depth} = \frac{2.33}{P} = \frac{2.33}{16} = .1456$$ Fig. 3-1

(10) Gear testing radius.

(i) Assume the master gear will have 30 teeth and the standard normal circular tooth thickness of .0982.

$$\text{Standard pitch diameter of master gear} = \frac{N}{P_n \times \cos\psi} = \frac{30}{16 \times .95105652} = 1.9715$$

(ii) Find the close mesh center distance of pinion and master for the maximum pinion tooth thickness of .1221.

$$C = \frac{(N_1 + N_2) \times \cos\phi}{2 \times P_n \times \cos\psi \times \cos\phi_1}$$ Equ. 9-11

$N_1 = 30 \quad N_2 = 15 \quad \cos\phi = .93394337 \quad P_n = 16 \quad \cos\psi = .95105652$

$$\text{inv}\,\phi_1 = \frac{P_n(t_{n1} + t_{n2}) - \pi}{N_1 + N_2} + \text{inv}\,\phi$$

$t_{n1} = .0982 \quad t_{n2} = .1221 \quad \text{inv}\,\phi = .01719592$

$$\text{inv}\,\phi_1 = \frac{16(.0982 + .1221) - 3.1415926}{30 + 15} + .01719592$$

$$= .02571164$$

$$\phi_1 = 23.813965°$$

$$\cos\phi_1 = .91486128$$

$$C = \frac{(30 + 15) \times .93394337}{2 \times 16 \times .95105652 \times .91486128}$$

$$= 1.5095$$

(iii) Repeat for minimum pinion tooth thickness of .1201.

$$\operatorname{inv}\phi_1 = \frac{16(.0982 + .1201) - 3.1415926}{30 + 15} + .01719592$$

$$= .0250053$$

$$\phi_1 = 23.602694°$$

$$\cos\phi_1 = .91634390$$

$$C = \frac{(30 + 15) \times .93394337}{2 \times 16 \times .95105652 \times .91634390}$$

$$= 1.5070$$

(iv) Find the close mesh center distance of gear and master for the maximum gear tooth thickness.

$$C = \frac{(N_1 + N_2) \times \cos\phi}{2 \times P_n \times \cos\psi \times \cos\phi_1}$$

$N_1 = 30$ $N_2 = 45$ $\cos\phi = .93394337$ $P_n = 16$ $\cos\psi = .95105652$

$$\operatorname{inv}\phi_2 = \frac{P_n(t_{n1} + t_{n2}) - \pi}{N_1 + N_2} + \operatorname{inv}\phi$$

$t_{n1} = .0982$ $t_{n2} = .0961$ $\operatorname{inv}\phi = .01719592$

$$\operatorname{inv}\phi_1 = \frac{16(.0982 + .0961) - 3.1415926}{30 + 45} + .01719592$$

$$= .01675868$$

$$\phi_1 = 20.769295°$$

$$\cos\phi_1 = .93501585$$

$$C = \frac{(30 + 45) \times .93394337}{2 \times 16 \times .95105652 \times .93501585}$$

$$= 2.4615$$

(v) Repeat for minimum tooth thickness of .0941.

$$\operatorname{inv}\phi_1 = \frac{16(.0982 + .0941) - 3.1415926}{30 + 45} + .01719592$$

$$= .01633202$$

$$\phi_1 = 20.597792$$

$$\cos\phi_1 = .93607309$$

$$C = \frac{(30 + 45) \times .93394337}{2 \times 16 \times .95105652 \times .93607309}$$

$$= 2.4588$$

(vi) Pitch diameter of master gear $= 1.9715$

Total composite tolerance of pinion $= .0043$

Max. testing radius of pinion $= 1.5095 + \dfrac{.0043}{2} - \dfrac{1.9715}{2}$

$= .5259$

Min. testing radius of pinion $= 1.5070 - \dfrac{.0043}{2} - \dfrac{1.9715}{2}$

$= .5191$

Total composite tolerance of gear $= .0047$

Max. testing radius of gear $= 2.4615 + \dfrac{.0047}{2} - \dfrac{1.9715}{2}$

$= 1.4781$

Min. testing radius of gear $= 2.4588 - \dfrac{.0047}{2} - \dfrac{1.9715}{2}$

$= 1.4707$

(11) Maximum tooth-to-tooth composite tolerance.

From the AGMA "Gear Handbook" it is found that the maximum tooth-to-tooth composite tolerances of the pinion and gear are .0021 and .0017 respectively.

(12) Testing pressure.

According to the AGMA "Gear Handbook" the testing pressure recommended for non-metallic gears having diametral pitches of 10-19 should range from 14.5 to 17.5 ounces. Let the testing pressure be 15 ounces.

(13) Measurement over two pins.

(i) Find measurement over two pins of pinion for maximum tooth thickness of .1221.

$$M = \left(\frac{N \times \cos\phi}{P_n \times \cos\psi \times \cos\phi_1}\right)\left(\cos\frac{90^\circ}{N}\right) + d \qquad \text{Equ. 9-17(b)}$$

$N = 15 \quad \cos\phi = .93394337 \quad P_n = 16 \quad \cos\psi = .95105652$

$\dfrac{90^\circ}{N} = \dfrac{90^\circ}{15} = 6^\circ \qquad \cos\dfrac{90^\circ}{N} = .99452189$

$\dfrac{1.728}{16} = .108 \qquad \text{Let } d = .110$

$$\text{inv}\,\phi_1 = \frac{P_n}{N}\left(t_n + \frac{d \times \cos\psi}{\cos\psi_1 \times \cos\phi}\right) + \text{inv.}\,\phi - \frac{\pi}{N}$$

$t_n = .1221 \qquad \text{inv}\,\phi = .01719592$

$\psi_1 = \tan^{-1}(\tan\psi \times \cos\phi)$

$= \tan^{-1}(.32491970 \times .93394337)$

$= 16.880767^\circ$

$$\cos\psi_1 = .95691112$$

$$\text{inv}\,\phi_1 = \frac{16}{15}\left(.1221 + \frac{.110 \times .95105652}{.95691112 \times .93394337}\right) + .01719592 - \frac{3.1415926}{15}$$

$$= .06285993$$

$$\phi_1 = 31.475236°$$

$$\cos\phi_1 = .85286591$$

$$M = \left(\frac{15 \times .93394337}{16 \times .95105652 \times .85286591}\right)(.99452189) + .110$$

$$= 1.1835$$

(ii) Repeat for minimum tooth thickness of .1201.

$$\text{inv}\,\phi_1 = \frac{16}{15}\left(.1201 + \frac{.110 \times .95105652}{.95691112 \times .93394337}\right) + .01719592 - \frac{3.1415926}{15}$$

$$= .06072660$$

$$\phi_1 = 31.144860°$$

$$\cos\phi_1 = .85586240$$

$$M = \left(\frac{15 \times .93394337}{16 \times .95105652 \times .85586240}\right)(.99452189) + .110$$

$$= 1.1798$$

Measurement over two (.110) pins = 1.1835 max., 1.1798 min.

(iii) Find measurement over two pins of gear for maximum tooth thickness of .0961.

$$M = \left(\frac{N \times \cos\phi}{P_n \times \cos\psi \times \cos\phi_1}\right)\left(\cos\frac{90°}{N}\right) + d$$ Equ. 9-17(b)

$N = 45$ $\quad \cos\phi = .93394337$ $\quad P_n = 16$ $\quad \cos\psi = .95105652$

$\frac{90°}{N} = \frac{90°}{45} = 2°$ $\quad \cos\frac{90°}{N} = .99939082$ $\quad d = .110$

$$\text{inv}\,\phi_1 = \frac{P_n}{N}\left(t_n + \frac{d \times \cos\psi}{\cos\psi_1 \times \cos\phi}\right) + \text{inv}\,\phi - \frac{\pi}{N}$$

$t_n = .0961$ $\quad \text{inv}\,\phi = .01719592$ $\quad \cos\psi_1 = .95691112$

$$\text{inv}\,\phi_1 = \frac{16}{45}\left(.0961 + \frac{.110 \times .95105652}{.95691112 \times .93394337}\right) + .01719592 - \frac{3.1415926}{45}$$

$$= .02317281$$

$$\phi_1 = 23.039268°$$

$$\cos\phi_1 = .92023685$$

$$M = \left(\frac{45 \times .93394337}{16 \times .95105652 \times .92023685}\right)(.99939082) + .110$$

$$= 3.1095$$

(iv) Repeat for minimum tooth thickness

$$\operatorname{inv}\phi_1 = \frac{16}{45}\left(.0941 + \frac{.110 \times .95105652}{.95691112 \times .93394337}\right) + .01719592 - \frac{3.1415926}{45}$$

$$= .02246170$$

$$\phi_1 = 22.811496$$

$$\cos\phi_1 = .92178538$$

$$M = \left(\frac{45 \times .93394337}{16 \times .95105652 \times .92178538}\right)(.99939082) + .110$$

$$= 3.1044$$

Measurement over two (.110) pins = 3.1095 max., 3.1044 min.

(14) Lead of pinion

$$L = \pi \times D \times \cot\psi$$

$$D = .9857 \qquad \psi = 18^\circ \qquad \cot\psi = 3.0776836$$

$$L = 3.1415926 \times .9857 \times 3.0776836$$

$$= 9.5310$$

(15) Lead of gear

$$L = \pi \times D \times \cot\psi$$ Equ. 8-1

$$D = 2.9572 \qquad \psi = 18^\circ \qquad \cot\psi = 3.0776836$$

$$L = 3.1415926 \times 2.9572 \times 3.0776836$$

$$= 28.5927$$

(16) Maximum root diameters of pinion and gear

$$D_R = \frac{1}{P_n}\left(\frac{N}{\cos\psi} - 6.9758\right) + (2.7475 \times t_n)$$

(i) Pinion

$$P_n = 16 \qquad N = 15 \qquad \cos\psi = .95105652 \qquad t_n = .1221$$

$$D_R = \frac{1}{16}\left(\frac{15}{.95105652} - 6.9758\right) + (2.7475 \times .1221)$$

$$= .8853$$

(ii) Gear

$$P_n = 16 \qquad N = 45 \qquad \cos\psi = .95105652 \qquad t_n = .0961$$

$$D_R = \frac{1}{16}\left(\frac{45}{.95105652} - 6.9758\right) + (2.7475 \times .0961)$$

$$= 2.7853$$

The data to go on the drawings are now complete and are listed as shown in Figs. 10-3 and 10-4.

FIG. 10-3

HELICAL GEAR DATA		
BASIC SPECIFICATIONS	NUMBER OF TEETH	15
	NORMAL DIAMETRAL PITCH	16
	NORMAL PRESSURE ANGLE	20°
	HELIX ANGLE	18.0000°
	HAND OF HELIX	RH
	STANDARD PITCH DIAMETER	.9857
	TOOTH FORM	PGT-1
	ADDENDUM	.0625
	WHOLE DEPTH	.1456
	CALC. NORMAL CIR. TOOTH THICKNESS ON STD. PITCH CIRCLE	.1221 MAX. .1201 MIN.
MANUFACTURING AND INSPECTION	GEAR TESTING RADIUS	.5259 MAX. .5191 MIN.
	AGMA QUALITY NUMBER	Q7
	MAXIMUM TOTAL COMPOSITE TOLERANCE	.0043
	MAXIMUM TOOTH-TO-TOOTH COMPOSITE TOLERANCE	.0021
	MASTER GEAR SPECIFICATIONS	$N = 30$ $t_n = .0982$
	TESTING PRESSURE (OUNCES)	15
	DIAMETER OF MEASURING PIN	.110
	MEASUREMENT OVER TWO PINS (FOR SETUP ONLY)	1.1835 MAX. 1.1798 MIN.
	LEAD	9.5310
	OUTSIDE DIAMETER	$1.177 {}^{+.000}_{-.006}$
	MAXIMUM ROOT DIAMETER	.8853
ENGINEERING REFERENCES	MATING GEAR PART NUMBER	GEAR
	NUMBER OF TEETH IN MATING GEAR	45
	OPERATING CENTER DISTANCE	2.0160 MAX. 2.0110 MIN.

FIG. 10-4

	HELICAL GEAR DATA	
BASIC SPECIFICATIONS	NUMBER OF TEETH	45
	NORMAL DIAMETRAL PITCH	16
	NORMAL PRESSURE ANGLE	20°
	HELIX ANGLE	18.0000°
	HAND OF HELIX	L.H.
	STANDARD PITCH DIAMETER	2.9572
	TOOTH FORM	PGT-1
	ADDENDUM	.0625
	WHOLE DEPTH	.1456
	CALC. NORMAL CIR. TOOTH THICKNESS ON STD. PITCH CIRCLE	.0961 MAX. .0941 MIN.
MANUFACTURING AND INSPECTION	GEAR TESTING RADIUS	1.4781 MAX. 1.4707 MIN.
	AGMA QUALITY NUMBER	Q7
	MAXIMUM TOTAL COMPOSITE TOLERANCE	.0047
	MAXIMUM TOOTH-TO-TOOTH COMPOSITE TOLERANCE	.0017
	MASTER GEAR SPECIFICATIONS	$N = 30$ $t_n = .0982$
	TESTING PRESSURE (OUNCES)	15
	DIAMETER OF MEASURING PIN	.110
	MEASUREMENT OVER TWO PINS (FOR SETUP ONLY)	3.1095 MAX. 3.1044 MIN.
	LEAD	28.5930
	OUTSIDE DIAMETER	$3.077 {+.000 \atop -.006}$
	MAXIMUM ROOT DIAMETER	2.7853
ENGINEERING REFERENCES	MATING GEAR PART NUMBER	PINION
	NUMBER OF TEETH IN MATING GEAR	15
	OPERATING CENTER DISTANCE	2.0160 MAX. 2.0110 MIN.

HORSEPOWER RATINGS

In the years that have intervened since "Accurate Molded Gears" was first published, nothing has transpired that would indicate any necessity to revise the horsepower formulas to be found there. In this chapter is presented a somewhat more sophisticated equation formulated specifically for use in estimating the load-carrying capacity of spur and helical gears, molded of the plastics and designed in conformity to the PGT Balanced Tooth Strength System.

Research and testing is continuing in an endeavor to increase the limited amount of knowledge now available about the behavior of plastics in gears. When more data are forthcoming, revisions to horsepower equations will probably be necessary. For that reason this chapter has been kept to the end of the section so that it might be the more easily replaced, as and when revisions are called for.

Because of the number of plastics, in various grades, now on the market, it is suggested that the technical staffs of the suppliers of the plastics under consideration for a gear application be consulted. It is desirable to obtain from them the most recent information in their possession about the load bearing capacities of gears molded of their particular plastics.

The fundamental horsepower equation for spur and helical gears molded of the plastics and designed to the PGT System is as follows:

Equation 11-1

$$HP = \frac{D \times F \times n \times J \times St \times K_T \times K_L}{126{,}000 \times P \times C_s \times K_R}$$

where:

HP = horsepower
D = operating pitch diameter
F = effective face width
n = speed, rpm
J = geometry factor
St = tensile strength of plastic
K_T = temperature factor
K_L = life factor
P = diametral pitch, or normal diametral pitch
C_S = service factor
K_R = factor of safety

(1) Operating pitch diameter (D)

The operating pitch diameter of a gear cannot be determined until it is meshed with its mate at a definite operating center distance. Knowing the operating center distance and the numbers of teeth in two mating gears, the operating pitch diameters are obtained by use of Equations 11-2 and 11-3. The gear having the lesser number of teeth is referred to as the pinion and its mate as the gear.

Equation 11-2

$$d = \frac{2 \times C}{mg + 1}$$

Equation 11-3

$$D = \frac{2 \times C \times mg}{mg + 1}$$

where:

d = operating pitch diameter of pinion
D = operating pitch diameter of gear
$mg = \text{ratio} = \dfrac{\text{number of teeth in gear}}{\text{number of teeth in pinion}}$
C = operating center distance

(2) Effective face width (F)

F = face width that is in contact with mating gear.

(3) Speed (n)

n = number of revolutions turned in one minute.

(4) Geometry factor (J)

The geometry factors of the four PGT tooth forms are given in Table 11-1

Table 11-1

Tooth form	PGT-1	PGT-2	PGT-3	PGT-4
Geometry factor (J)	0.75	0.65	0.60	0.55

(5) Tensile strength of plastic (St)

The tensile strength of the plastic may be obtained from published properties charts or, preferably, directly from the technical staff of the supplier of the specific plastic chosen for the application.

(6) Temperature factor (K_T)

The temperature factor allows for the decrease in the tensile strength of a plastic with increase in temperature.

Equation 11-4

$$K_T = 1.0 - \left[(T - 70) \times .003 \right]$$

where:

K_T = temperature factor

T = maximum temperature to which gears will be subjected (° F).

(7) Life factor (K_L)

The life factor adjusts the horsepower rating for the number of cycles required of the gear before it fails.

Equation 11-5

$$K_L = 1.0 - \left(\frac{\log M}{5} \right)$$

where:

K_L = life factor

$$M = \frac{n \times \text{hours to failure} \times 60}{1{,}000{,}000}$$

(8) Diametral pitch, or normal diametral pitch (P)

P is the diametral pitch of a spur gear, or normal diametral pitch of a helical gear.

(9) Service factor (C_s)

The service factor takes into account the nature of the load on the mechanism driven by the gears. The appropriate value is obtained from Table 11-2

Table 11-2

Type of load	8-10 hours per day	24 hours per day	Intermittent 1-3 hours per day
Steady	1.00	1.25	0.80
Light shock	1.25	1.50	1.00
Medium shock	1.50	1.75	1.25
Heavy shock	1.75	2.00	1.50

(10) Factor of safety (K_R)

The value given to the factor of safety can range from 1.0 to 2.0, or greater. It could be said to vary in inverse proportion to the extent to which the designer has found it possible to adhere to the precepts set forth in the preceding chapters. If the gears have an adequate contact ratio and a high percentage of recess action; if they are designed as advocated for a power drive, and have teeth of balanced strength; if some degree of lubrication exists, and if the designer has complete confidence in the data relating to the properties of the plastics of which it is proposed to mold the gears, then there is no reason why the factor of safety should be greater than 1.0. If a high degree of reliability is desired it could be increased to 1.25 or, possibly, 1.33.

On the other hand, if the designer has found it to be necessary to make compromises because of circumstances beyond his or her control, then the extent to which the gears have been weakened must be assessed and the factor of safety increased accordingly.

As a last word on the load carrying capacities of gears molded of the plastics, it might be pointed out that while production gears molding dies can be expensive, prototype gears for testing can be produced from single cavities in universal frames at quite reasonable prices. Much of the cost is recoverable when the production molds come to be made. No amount of theorizing is a substitute for test-running prototypes in the mechanism they will be required to drive, and under the conditions to which they will be subjected in the field.

Example 11-1

The pair of helical gears in Example 10-2 are designed for a speed reducing drive in a domestic appliance. The pinion and gear have 15 and 45 teeth respectively, and a normal diametral pitch of 16. The PGT-1 tooth form is specified and the teeth are designed to have balanced strength. The operating center distance is 2.0160 max., 2.0110 min. The pinion has a face width of .980±.005 and the face width of the gear is .800±.005.

The pinion rotates at a speed of 1745 rpm. The drive operates intermittently for one or two hours per day and is subject to light shock loading. A life of 3,000 hours is required. The gears are grease lubricated. The pinion is molded of a plastic having a tensile strength of 10,000 psi and the gear of a plastic having a tensile strength of 9,000 psi.

Determine horsepower rating of pinion

$$HP = \frac{D \times F \times n \times J \times St \times K_T \times K_L}{126{,}000 \times P \times C_s \times K_R} \qquad \text{Equ. 11-1}$$

(1) Operating pitch diameter (D)

$$d = \frac{2 \times C}{mg + 1} \qquad \text{Equ. 11-2}$$

$$C = \frac{2.0160 + 2.0110}{2} = 2.0135$$

$$mg = \frac{45}{15} = 3$$

$$d = \frac{2 \times 2.0135}{3 + 1} = 1.0068$$

Because D in Equation 11-1 is, in this instance, the operating pitch diameter of the pinion, it is given the value obtained from Equation 11-2.

$$D = 1.0068$$

(2) Effective face width (F)

$$F = .800$$

(3) Speed (n)

$$n = 1745$$

(4) Geometry factor (J)

From Table 11-1

$$J = .75$$

(5) Tensile strength of plastic (St)

$$St = 10{,}000$$

(6) Temperature factor (K_T)

$$K_T = 1.0 - \left[(T - 70) \times .003\right] \qquad \text{Equ. 11-4}$$

$$T = 175$$

$$K_T = 1.0 - \left[(175 - 70) \times .003\right]$$

$$K_T = .685$$

(7) Life factor (K_L)

$$K_L = 1.0 - \left(\frac{\log M}{5}\right) \qquad \text{Equ. 11-5}$$

$$M = \frac{n \times \text{hours to failure} \times 60}{1{,}000{,}000}$$

$n = 1745$ hours to failure $= 3{,}000$

$$M = \frac{1745 \times 3000 \times 60}{1{,}000{,}000} = 314$$

$$K_L = 1.0 - \left(\frac{\log 314}{5}\right) = .50$$

(8) Diametral pitch, or normal diametral pitch (P)

$$P = 16$$

(9) Service factor (C_s)

From Table 11-2

$$C_s = 1.0$$

(10) Factor of safety (K_R)

The pinion and gear are designed for a power drive and have balanced tooth strength. The contact ratio is 2.48 and there is 72 percent recess action. A degree of lubrication is provided. A factor of safety of, say, 1.2 would be adequate for good reliability.

$$K_R = 1.2$$

$$HP\text{ (pinion)} = \frac{1.0068 \times .800 \times 1745 \times .75 \times 10{,}000 \times .685 \times .50}{126{,}000 \times 16 \times 1.0 \times 1.2} = 1.5$$

Determine horsepower rating of gear

$$HP = \frac{D \times F \times n \times J \times St \times K_r \times K_L}{126{,}000 \times P \times C_s \times K_R} \qquad \text{Equ. 11-1}$$

(1) Operating pitch diameter (D)

$$D = \frac{2 \times C \times mg}{mg + 1} = \frac{2 \times 2.0135 \times 3}{3 + 1} = 3.0202 \qquad \text{Equ. 11-3}$$

(2) Effective face width (F)

$$F = .800$$

(3) Speed (n)

$$n = \frac{1745}{3} = 582$$

(4) Geometry factor (J)

$$J = .75$$

(5) Tensile strength of plastic (St)

$$St = 9{,}000$$

(6) Temperature factor (K_T)

$$K_T = .685$$

(7) Life factor (K_L)

$$K_L = 1.0 - \left(\frac{\log M}{5}\right) \qquad \text{Equ. 11-5}$$

$$M = \frac{314}{3} = 105$$

$$K_L = 1.0 - \left(\frac{\log 105}{5}\right) = .60$$

(8) Diametral pitch, or normal diametral pitch (P)

$$P = 16$$

(9) Service factor (C_s)

$$C_s = 1.0$$

(10) Factor of safety (K_R)

$$K_R = 1.2$$

$$HP\ (\text{gear}) = \frac{3.0202 \times .800 \times 582 \times .75 \times 9{,}000 \times .685 \times .60}{126{,}000 \times 16 \times 1.0 \times 1.2} = 1.6$$

The horsepower rating of the pinion is less than that of the gear and establishes the rating of the drive.

Horsepower rating of the drive = 1.5

CROWNED MOLDED PLASTICS GEARS

A NEW SOLUTION TO AN OLD PROBLEM

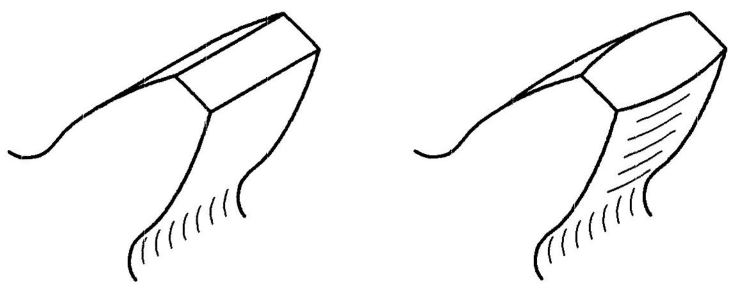
CONTENTS

EDITOR'S NOTE: Many of the sketches, especially those depicting misalignments, are exaggerated to clarify a point.

INTRODUCTION

The old problem is the common condition of misaligned mating parallel-axis (spur and helical) gear tooth surfaces, as described below. The solution, new to molded plastic gears but old in machined gear practice, is the modification of gear flanks by making them full thickness at mid-face-width and tapering them to each edge. See figure (1). This modification, extending over the full height of the gear tooth, is referred to as *crowning*. The result is a *crowned* tooth or gear, not to be confused with other references in gear terminology.

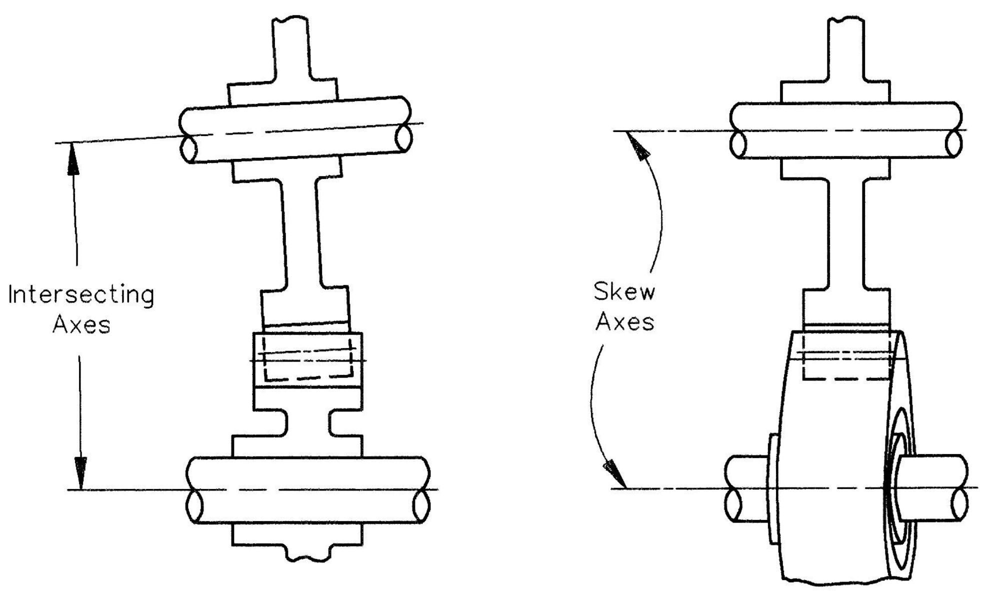
Fig (1)



MISALIGNMENT of GEAR TEETH

Instead of the ideally parallel gear rotation axes, the axes may be intersecting in the same plane, or skew in different planes, or a combination of both. See figure (2) for illustrations relating to spur gears.

Fig (2)

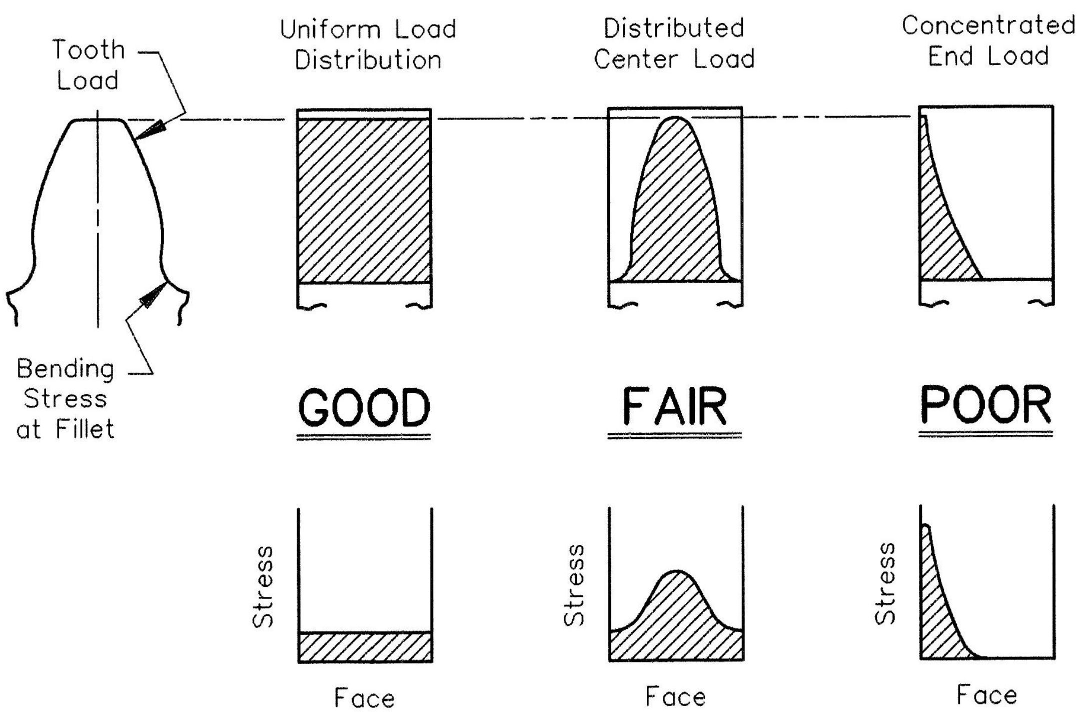


PROBLEMS CAUSED by MISALIGNMENT

The major problem associated with gear tooth misalignment is the concentration of contact to a narrow strip at one end of the gear face width. The load transmitted through this narrow width gains support from adjacent material on only one side of the contact area, resulting in high bending stresses. See figure (3). These high stresses may lead to the start of a crack which progresses across the face width until a major portion of the tooth breaks. Also, the concentrated area of contact may initiate rapid tooth surface wear with the wear detritus further accelerating surface failure.

In the case of helical gears, the localized area of contact due to misalignment reduces the helical continuity of contact. This reduction effectively eliminates the 'smoothing' action in motion transmission expected from helical gears.

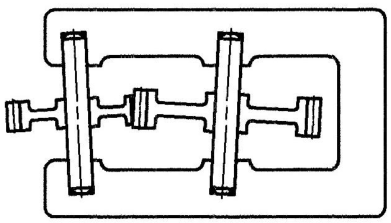
Fig (3)



SOURCES of MISALIGNMENT

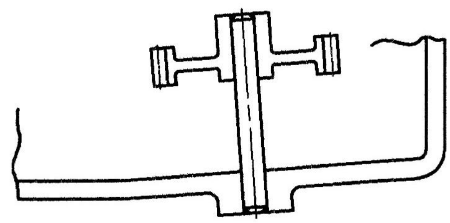
Molded plastic gears are commonly used with molded plastic housings, often of a fiber reinforced material. This process, with its often unpredictable distortions, makes it difficult to achieve and maintain a high degree of accuracy in gear alignment features. The same may be true in die-cast metal housings, even if less so with secondary machined bearing openings. All these housings are subject to further distortions under changes in temperature or the passage of time. See figures (4a) (4b).

Fig (4a)



Housing Misalignment

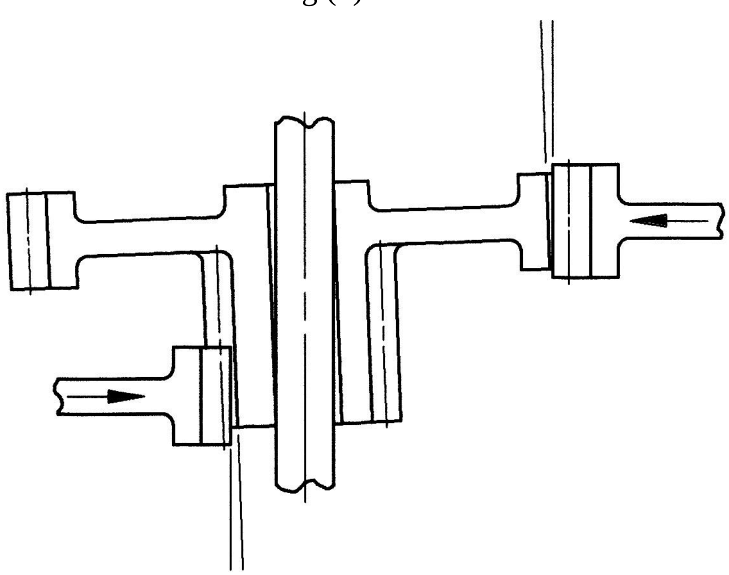
Fig (4b)



Housing Distortion

Also, molded plastic gears are most likely to be supported in sleeve bearings with relatively large clearances in contrast to the small clearances in ball bearings. If the journal and bearing diameters are produced by molding, with their sizes controlled by typical tolerances, the resulting clearances may permit significant misalignment. As shown in figure (5), the misalignment results from the combination of the clearance and the axially offset and opposing forces, such as those on compound gears. .

Fig (5)



Offset Tooth Loading on Driving & Driven Gears

In some plastic gear assemblies, the gears are supported by small diameter metal shafts, either fixed to the gear and rotating, or fixed to the housing and non-rotating. With either simple or cantilevered types of supports, there are further opportunities for gear misalignment. This results when there are forces large enough to deflect the shafts and when the gears are positioned on the high slope portion of the shaft deflection curve, as shown in figures (6a) and (6b). With slender molded plastic shafts in place of metal, the deflections may be even greater. Cantilevered support may also be accompanied by a compliant structure, as shown in figure (6c), where it is shown without accompanying shaft deflection.

Fig (6a)

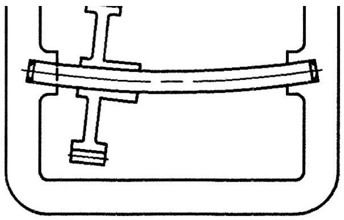


Fig (6b)

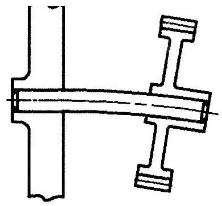
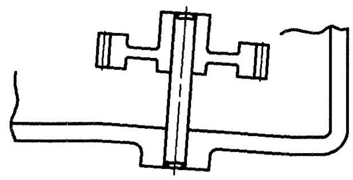
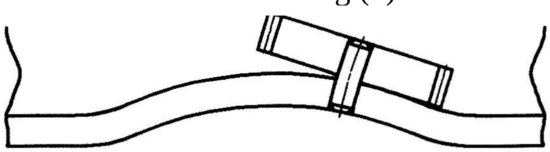


Fig (6c)



One type of gear assembly is particularly sensitive to misalignment due to deflections under load. The planet gear in planetary gear arrangements is often supported by a cantilevered shaft, which may be of limited diameter to reduce bearing friction losses. The web portion of the planet carrier, connecting the planet shafts, may be of reduced thickness to save space. The combined deflection of these features under load, pictured in figure (7) contributes to gear mesh misalignment.

Fig (7)



Then the gears themselves may have tapered faces, figure (8a), which will misalign and/or tapered bores which will allow movement under load. Distortion in the web creates wobble of the gear teeth to both sides of center, figure (8b).

Fig (8a)

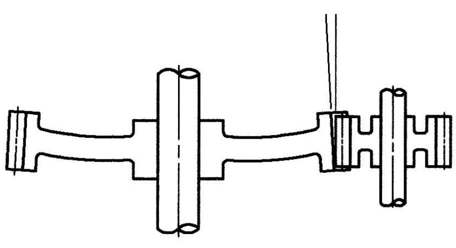
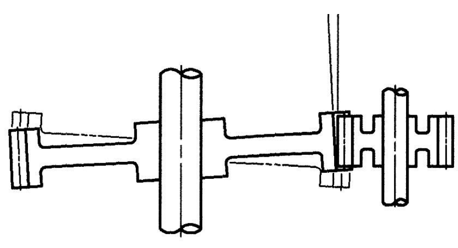


Fig (8b)



BENEFITS of CROWNING

The use of crowned gears can improve the adverse conditions imposed by tooth misalignment. Contact will be shifted away from the end of the tooth to some central location along the tooth flank. Instead of contact on a nearly sharp edge, over a narrow width at best, contact will take place along a gradually curved surface over a greater width. Support to the applied tooth load will come from this greater contact width and also from adjoining material on both sides of the contact center. Wear at the broader contact area will progress more slowly. If helical, more of the helical gear action will be maintained, often preserving the helical gear noise reduction. Refer back to figure (3).

There may be a further, if indirect, benefit of crowning. The application of crowning will permit a greater tolerance of misalignment in the product assembly. This relief may often be converted into manufacturing cost reduction.

CROWNING of MACHINED GEARS

There is a long standing practice of crowning machined steel gears. The modification of metal tooth surfaces is generally accomplished by secondary operations.

Crowned gears have been used in a great variety of gear transmissions, including automotive. There is a need even when the transmission housing is of rigid metal construction with accurately machined features for mounting ball bearings with negligible clearances. In applications associated with molded plastic gears, the need is greater.

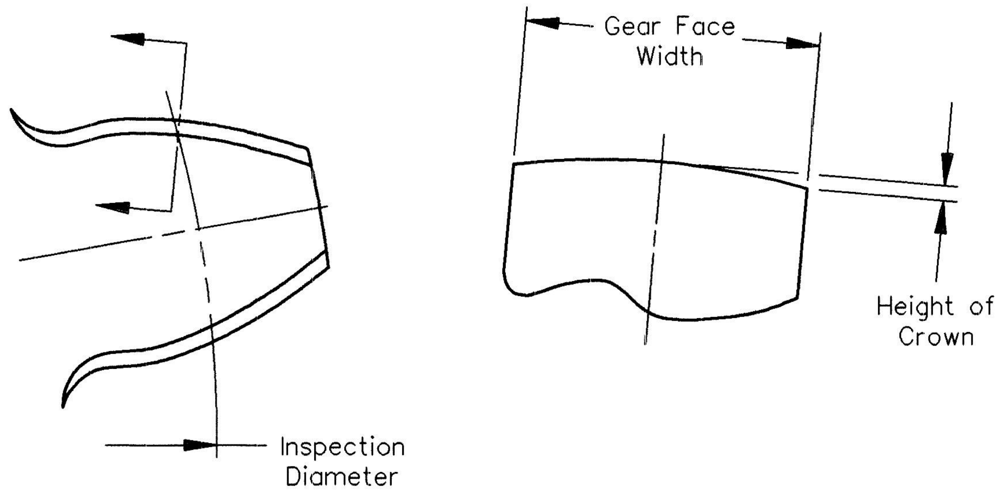
CROWNING of MOLDED PLASTICS GEARS

The recent introduction of crowned molded plastic gears has required significant development in new tooling and processing methods. The new tooling includes the construction of the mold cavity with varying cross-section, smallest at the ends and largest at the center of the face width The new processing covers the ejection of the molded gear while preserving the modified tooth surfaces. This requires the optimum control of the ejection timing so as to take advantage of the initial shrinkage and the limited elasticity of the still hot plastic material. This researched process is readily available for spur gears and some helical gears. Except for a moderate increase in the cost of tool construction, there is no significant increase in molded gear cost.

DESIGN of CROWNED GEARS

Crown is commonly specified by the height of the circular arc spanning the width of the gear tooth in a direction perpendicular to the tooth surface, see figure 9.

Fig (9)



A simple equation may be used to determine the height of crown needed:

$$h_C = \frac{2 F \alpha}{3}$$

where: h_C = height of the crown as a circular arc normal to the tooth surface, required to maintain contact in the central third of the face width, see figure 9

F = face width spanned by the crown

α = angle of misalignment, in radians

As an example, for shaft support misalignment of .005 inches over a length of 1.25 inches, giving a value to the angle, α, of .005 ÷ 1.25 = .004 radians. For a face width, F = .375 inches, the required height of crown, h_C = 2 x .375 (.004) ÷ 3 = .0010 inches

A tolerance, Tol(h_C), should be applied as an addition to the design value of crown height. A proposed tolerance may be calculated from the equation:

$$Tol(h_C) = .0002 + (.20 \times h_C) \text{ inches}$$

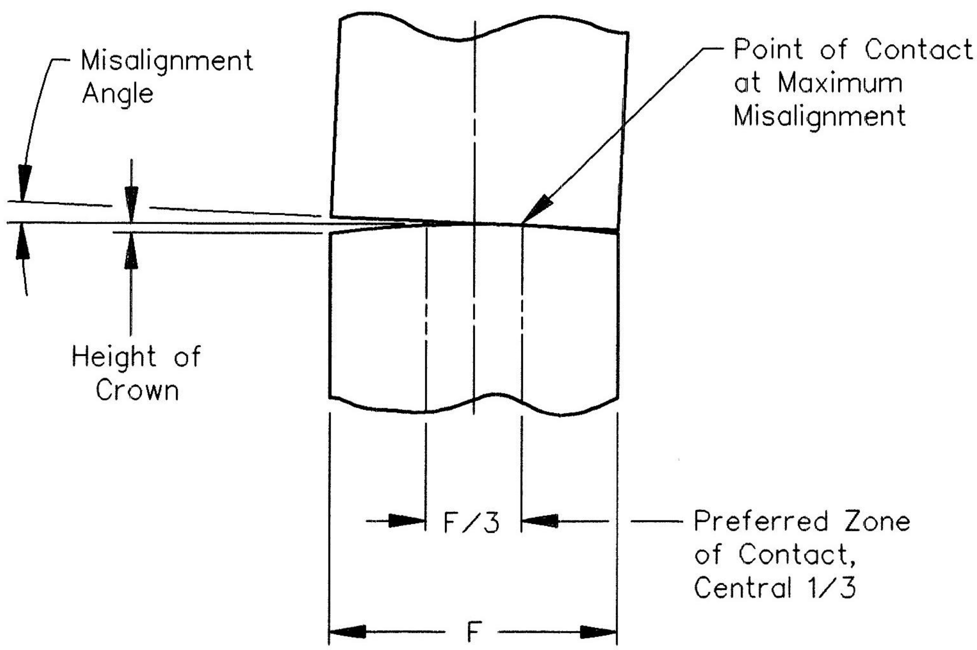
For the above example, the tolerance would be:

$$Tol(h_C) = .0002 + (.20 \times .0010) = .0004 \text{ inches}$$

The full required crown is generally applied to only one of the two mating gears, preferably to the gear which will provide the greater shrinkage clearance to assist in the ejection of the molded part. If the two gears are from materials with similar shrink rates, this would be the larger gear. If the required crown height is too large to be accommodated in the molding of a single gear, it may be divided as needed between the two mating gears.

The preferred zone of contact is the central one third of the face width. See figure (10).

Fig (10)



SIZE and PITCH LIMITATIONS

If the gears have very narrow face widths, there is no significant benefit from adding crowning. A tentative lower limit on face width in inches is 2.5 divided by the diametral pitch (in millimeters, 2.5 times the module). There is no upper limit, with the larger face width most likely to require greater crowning.

As to gear diameter, there is again no upper limit. The lower limit is based on the shrink rate of the gear material and other factors which will influence the ease of molded part ejection. For higher shrinkage material such as un-reinforced acetal or nylon, one-half inch (12.7 mm) pitch diameter is a tentative lower limit. This lower limit is likely to be revised as experience is expanded.

The pitch (or module) of the gear has to be considered before specifying crowning. There is no upper limit on how coarse the pitch. When it comes to how fine the pitch, restrictions apply that relate to the methods of mold cavity processing and, in some cases, to the fillet design of the gear tooth. A tentative limit is the diametral pitch of 32 (or the module of .80). This limit, too, is likely to be revised with greater experience.

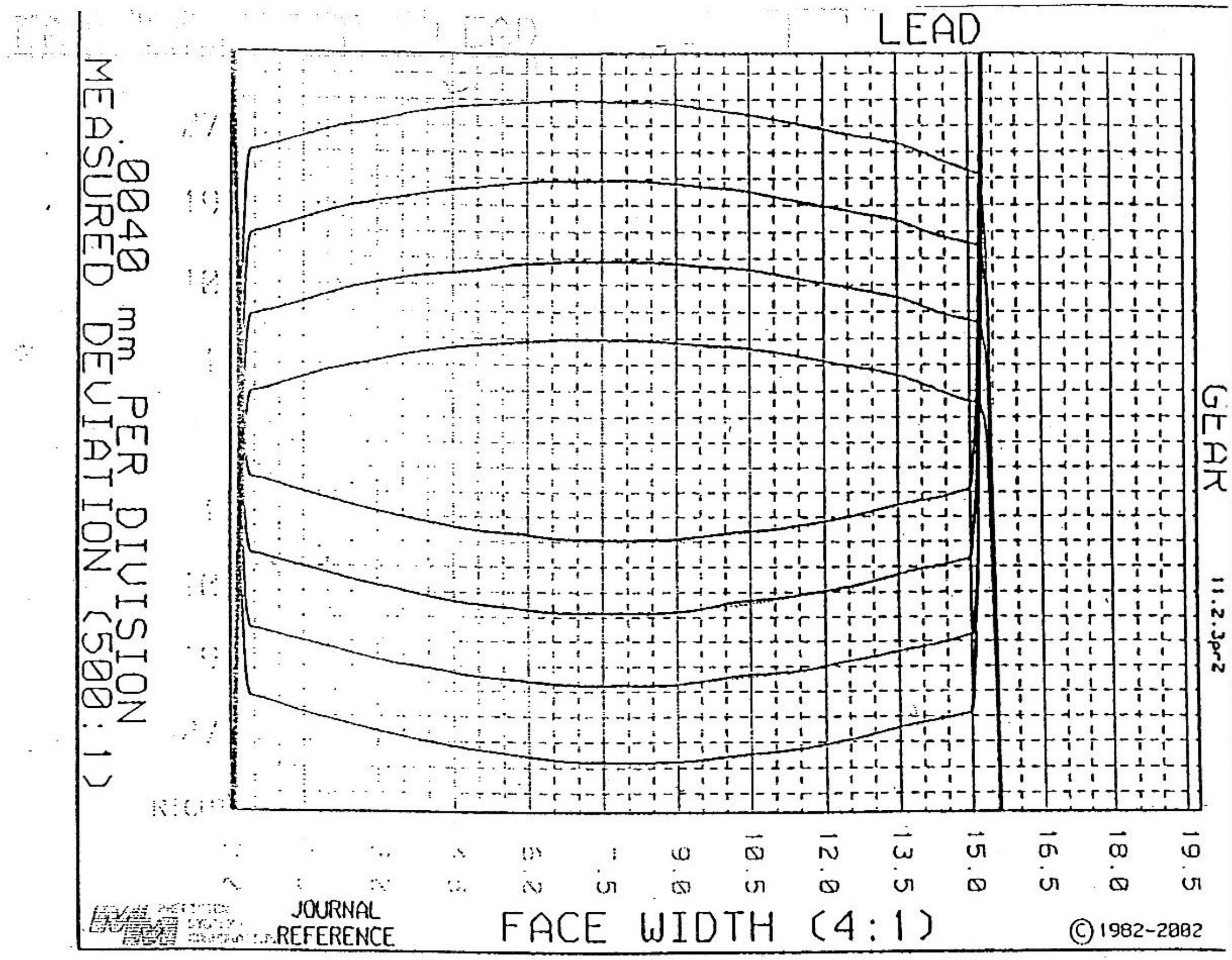
These limits are based on experience with spur gears. They may be applied to helical gears with helix angles up to, say, 30 degrees. However, other issues relating to tool processing and part ejection may, with further experience, introduce tighter limits.

CROWN INSPECTION

Inspection of the crown, for shape and height, is generally made on elemental gear inspection equipment, such as supplied by the M & M Company. The measuring probe follows the gear surface at the inspection diameter from one face to the other. On spur gears, its path is a straight line. On helical gears, the gear is rotated as specified by the gear design and the path of the probe relative to the rotating gear is a helix. This is typically applied to four equally, or nearly equally, spaced teeth.

The results of this inspection are in the form of magnified plots of the measured surfaces. The shape of the crown is apparent in each of these plots and the height may be read from the enlarged scale. In addition, the height values are typically supplied in printed form. See figures

Figure (11) represents measurements made on machined steel automotive gears. The printed values show crown heights of approximately .0003 inches (.007 mm).
Figure (12) represents measurements made on a plastics gear molded at ABA-PGT.
The values range from .0014 to .0017 inches, which handily met the original tolerance specification of .0015 +/- .0005 inches.



LEFT FLANK

TOOTH	1	10	19	27	TOL
TOTAL ERROR Fβ	.0016	.0016	.0022	.0029	.0090
ANGLE ERROR fHβ	-.0015	-.0012	-.0021	-.0032	.0080
FORM ERROR fβf	.0004	.0005	.0006	.0008	.0055
CROWN ACTUAL Cba	.0064	.0066	.0068	.0070	

MEAN

.0067 mm = .0003

RIGHT FLANK

TOOTH	1	10	19	27	TOL
TOTAL ERROR Fβ	.0015	.0011	.0009	.0016	.0090
ANGLE ERROR fHβ	.0011	-.0000	.0005	.0017	.0080
FORM ERROR fβf	.0007	.0009	.0009	.0005	.0055
CROWN ACTUAL Cba	.0071	.0073	.0067	.0068	

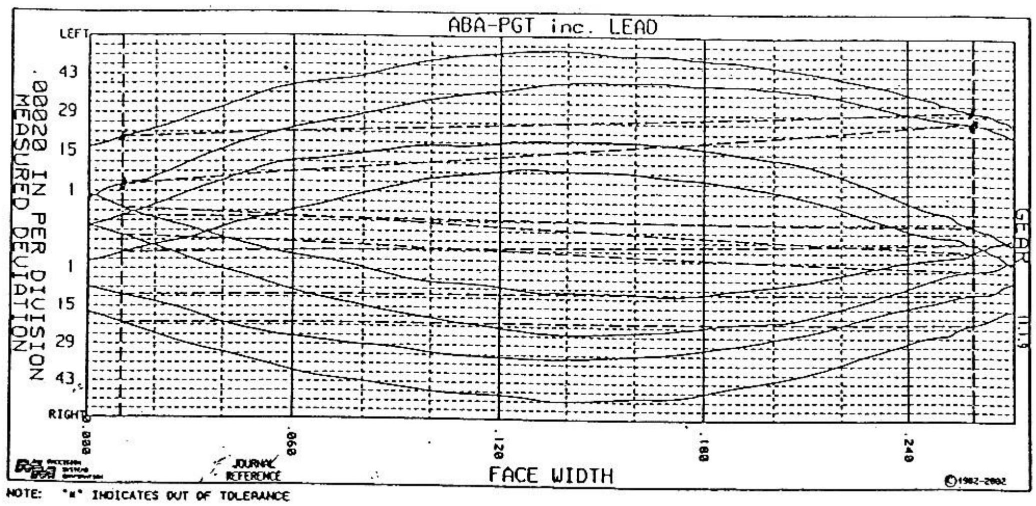
.0070 mm = .0003

Figure (11) Inspected crown on a machined steel automotive gear

OPERATOR : GLENN ELLIS
PART NAME : GEAR
INSPECTION NOTES :
.031 PROBE, .532 PIN, KO UP, .70 BLOCK GEAR FACE

DATE : 15 Sep 2003
TIME : 09:45

PART # : FP-01140 JR GEAR
SERIAL # : 2



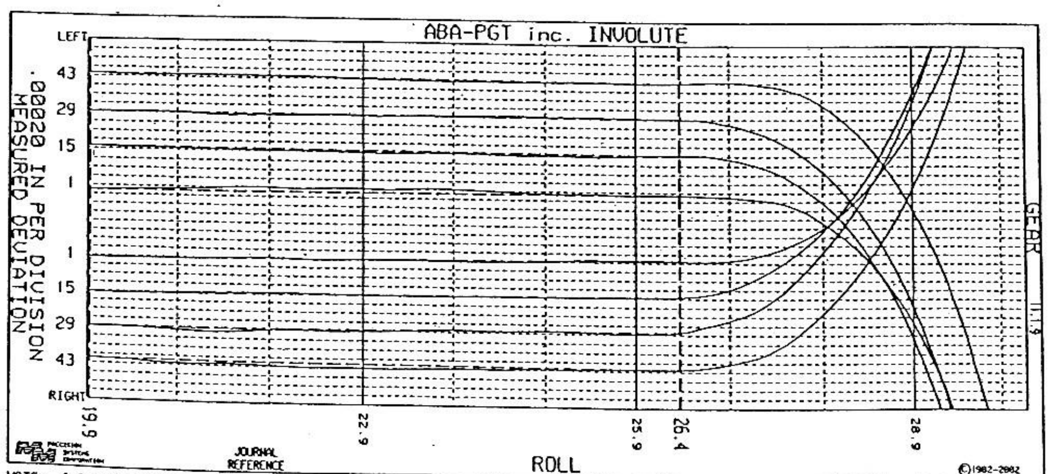
NOTE: "*" INDICATES OUT OF TOLERANCE

	LEFT FLANK		
TOOTH	SLOPE	HOLLOW	CROWN
1	.00024	.00000	.00162
15	-.00016	.00000	.00163
29	.00133	.00000	.00141
43	.00060	.00000	.00152

	RIGHT FLANK		
TOOTH	SLOPE	HOLLOW	CROWN
1	.00086	.00000	.00133
15	.00068	.00000	.00171
29	-.00008	.00000	.00138
43	-.00003	.00000	.00166

CONSTRUCTION LINE TYPE ANALYSIS

	LEFT FLANK	RIGHT FLANK
AVERAGE SLOPE	.00050	.00036
AVERAGE CROWN	.00155	.00152
MAXIMUM HOLLOW	.00000	.00000
MAX SLOPE VAR	.00149	.00094



NOTE: "*" INDICATES OUT OF TOLERANCE

	LEFT FLANK		
TOOTH	SLOPE	HOLLOW	CROWN
1	.00004	.00000	.00011
15	-.00005	.00005	.00000
29	-.00003	.00004	.00003
43	-.00008	.00004	.00001

	RIGHT FLANK		
TOOTH	SLOPE	HOLLOW	CROWN
1	-.00007	.00002	.00003
15	-.00006	.00004	.00002
29	-.00005	.00002	.00008
43	.00006	.00000	.00009

CONSTRUCTION LINE TYPE ANALYSIS

	LEFT FLANK	RIGHT FLANK
AVERAGE SLOPE	-.00003	-.00003
AVERAGE CROWN	.00004	.00005
MAXIMUM HOLLOW	.00005	.00004
MAX SLOPE VAR	.00012	.00014

Figure (12) Inspected crown on a plastics gear molded at ABA-PGT